

Unit 2 Test 3 Preview 2015

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$$1) \quad {}^6C_3 \times {}^4C_2 = 120$$

ii) 3 men + 2 women OR 4 men + 1 woman OR 5 men

$$\begin{aligned} & {}^6C_3 + {}^4C_2 & & {}^6C_4 + {}^4C_1 & & {}^6C_5 \\ & = 120 & & + 60 & & + 6 \\ & = 186 \end{aligned}$$

$$\text{iii) } \binom{1}{C_1} \times \binom{3}{C_1} \times \binom{5}{C_3} + \binom{3}{C_2} \times \binom{5}{C_2} \times \binom{1}{C_1} + \binom{3}{C_2} \times \binom{5}{C_3}$$

choose woman
omit man choose man
omit woman choose
neither

$$30 + 30 + 30 = 90$$

$$2) \quad P(M) = \frac{206}{427}$$

$$\text{ii) } P(M \text{ and } E) = \frac{412}{1281}$$

$$\text{iv) } \frac{179}{282}$$

$$3) a) \quad \frac{dy}{dx} + 2y = x$$

Auxiliary equation

$$u + 2 = 0$$

$$u = -2$$

$$y = Ae^{-2x}$$

$$\text{let } y = bx + c$$

$$\frac{dy}{dx} = b$$

$$b + 2(bx + c) = x$$

$$2bx + b + 2c = x$$

$$2b = 1$$

$$b = \frac{1}{2}$$

$$b + 2c = 0$$

$$c = -\frac{1}{4}$$

$$y = \frac{1}{2}x - \frac{1}{4}$$

$$y = Ae^{-2x} + \frac{1}{2}x - \frac{1}{4}$$

$$1 = Ae^{-2(0)} + \frac{1}{2}(0) - \frac{1}{4}$$

$$\frac{5}{4} = A$$

$$y = \frac{5}{4}e^{-2x} + \frac{1}{2}x - \frac{1}{4}$$

$$b) \frac{dy}{dx} = -\frac{5}{2}e^{-2x} + \frac{1}{2}$$

$$0 = -\frac{5}{2}e^{-2x} + \frac{1}{2}$$

$$-\frac{1}{2} = -\frac{5}{2}e^{-2x}$$

$$\frac{1}{5} = e^{-2x}$$

$$\ln(0.2) = -2x$$

$$-\frac{1}{2} \ln(0.2) = x$$

$$y = \frac{5}{4}e^{-2\left(-\frac{1}{2} \ln(0.2)\right)} + \frac{1}{2}\left(-\frac{1}{2} \ln(0.2)\right) - \frac{1}{4}$$

$$= \frac{5}{4}(0.2) - \frac{1}{4} \ln(0.2) - \frac{1}{4}$$

$$= -\frac{1}{4} \ln(0.2)$$

$$\left(-\frac{1}{2} \ln(0.2), -\frac{1}{4} \ln(0.2)\right)$$

$$4) z = xy$$

$$\frac{dz}{dx} = 1(y) + x \frac{dy}{dx}$$

$$\frac{d^2z}{dx^2} = \frac{dy}{dx} + 1 \frac{dy}{dx} + x \frac{d^2y}{dx^2}$$

$$\frac{d^2z}{dx^2} + 4z = 8x^2 + 16$$

$$2 \frac{dy}{dx} + x \frac{d^2y}{dx^2} + 4xy = 8x^2 + 16$$

Auxiliary equation

$$\rightarrow u^2 + 4 = 0$$

$$u^2 = -4$$

$$u = \pm 2i$$

$$z = a \cos 2x + b \sin 2x$$

$$\text{Let } z = cx^2 + dx + e$$

$$\frac{dz}{dx} = 2cx + d$$

$$\frac{d^2z}{dx^2} = 2c$$

$$2c + 4cx^2 + 4dx + 4e = 8x^2 + 16$$

$$4c = 8$$

$$c = 2$$

$$4d = 0$$

$$d = 0$$

$$2c + 4e = 16$$

$$4e = 12$$

$$e = 3$$

$$z = 2x^2 + 3$$

$$z = a \cos 2x + b \sin 2x + 2x^2 + 3$$

$$xy = a \cos 2x + b \sin 2x + 2x^2 + 3$$

$$y = \frac{a}{x} \cos 2x + \frac{b}{x} \sin 2x + 2x + \frac{3}{x}$$

$$\frac{dy}{dx} = -\frac{a}{x^2} \cos 2x - \frac{2a}{x} \sin 2x - \frac{b}{x^2} \sin 2x + \frac{2b}{x} \cos 2x + 2 - \frac{3}{x^2}$$

when $y=0$ $x = \frac{\pi}{2}$

$$0 = \frac{2a}{\pi} \cos \pi + \frac{2b}{\pi} \sin \pi + \pi + \frac{b}{\pi}$$

$$0 = -\frac{2a}{\pi} + \pi + \frac{b}{\pi}$$

$$\frac{2a}{\pi} = \frac{\pi^2 + b}{\pi}$$

$$2a = b + \pi^2$$

$$a = \frac{b + \pi^2}{2}$$

when $\frac{dy}{dx} = -2$ $x = \frac{\pi}{2}$

$$-2 = \left(\frac{b + \pi^2}{2}\right) \left(\frac{4}{\pi^2}\right) \cos(\pi) + \frac{4b}{\pi} \cos \pi + 2 - \frac{12}{\pi^2}$$

$$-2 = \frac{4(b + \pi^2)}{2\pi^2} - \frac{4b}{\pi} + 2 - \frac{12}{\pi^2}$$

$$\frac{4b}{\pi} = 4 - \frac{12}{\pi^2} + \frac{2(b + \pi^2)}{\pi^2}$$

$$= \frac{4\pi^2 - 12 + 12 + 2\pi^2}{\pi^2}$$

$$= \frac{6\pi^2}{\pi^2} = 6$$

$$b = \frac{3}{2} \pi$$

$$y = \frac{b + \pi^2}{2x} \cos 2x + \frac{3\pi}{2x} \sin 2x + 2x + \frac{3}{x}$$

5) $\begin{vmatrix} 1 & 1 & -3 \\ 3 & -1 & -1 \\ 5 & -3 & k \end{vmatrix}$

$$= 1 \begin{vmatrix} -1 & -1 \\ -3 & k \end{vmatrix} - 1 \begin{vmatrix} 3 & -1 \\ 5 & k \end{vmatrix} - 3 \begin{vmatrix} 3 & -1 \\ 5 & -3 \end{vmatrix}$$

$$= -k - 3 - 1(3k + 5) - 3(-9 + 5)$$

$$= -k - 3 - 3k - 5 + 12$$

$$= -4k + 4$$

$$\begin{array}{rclcl} 20x + 40y + 60z & = & 1120 & \div 20 \\ 40x + 60y + 80z & = & 1720 & \div 20 \\ 60x + 80y + 120z & = & 2480 & \div 20 \end{array}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 56 \\ 86 \\ 124 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 56 \\ 2 & 3 & 4 & 86 \\ 3 & 4 & 6 & 124 \end{array} \right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 56 \\ 0 & -1 & -2 & -26 \\ 0 & -2 & -3 & -44 \end{array} \right) \begin{array}{l} R_2 - 2R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 56 \\ 0 & 1 & 2 & 26 \\ 0 & -2 & -3 & -44 \end{array} \right) -R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 2 & 26 \\ 0 & 0 & 1 & 8 \end{array} \right) \begin{array}{l} R_1 - 2R_2 \\ R_3 + 2R_2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 8 \end{array} \right) \begin{array}{l} R_1 + R_3 \\ R_2 - 2R_3 \end{array}$$