Unit 2 Test 1 (PREVIEW)

1. Determine $\frac{\partial f}{\partial y}$ in each of the following cases.

   (i) $f(x,y) = 2x^3y + e^{2x+y} - \cos(xy)$ $\quad [2x^3 + e^{2x+y} + x\sin(xy)]$

   (ii) $f(x,y) = \ln(x^2y^2)\cos^{-1}(xy^2)$ $\quad \left[\frac{2\cos^{-1}(xy^2)}{y} + \frac{2xy\ln x^2y^2}{\sqrt{1-(xy^2)^2}}\right]$

2. The complex number, z, is such that $z = 4 - 3i$. Determine

   (i) $|z|$ $\quad [5]$

   (ii) $\arg z$ $\quad [-0.6435]$

   (iii) $\frac{z}{5-12i}$ $\quad \left[\frac{56}{169} + \frac{33}{169}i\right]$

3. Given that for $z = \cos\theta + i\sin\theta$,

$$z^n + \frac{1}{z^n} = 2\cos n\theta \quad \text{and} \quad z^n - \frac{1}{z^n} = 2i\sin n\theta$$

by considering $\left(z - \frac{1}{z}\right)^4\left(z + \frac{1}{z}\right)^2$, show that

$$\sin^4\theta\cos^2\theta = \frac{1}{32}(\cos 6\theta - 2\cos 4\theta - \cos 2\theta + 2)$$

Hence, find the exact value of

$$\int_0^{\frac{\pi}{4}} \sin^4\theta\cos^2\theta \; d\theta$$

$\left[\frac{3\pi - 4}{192}\right]$

4. Given that $\sec y = x$, for $x > 0$. Show that

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$$

5. A curve has parametric equation

$$x = 2\tan t, \quad y = 2\cos^2 t, \quad 0 < t < \frac{\pi}{2}$$

   (a) Find an expression for $\frac{dy}{dx}$ in terms of t. $\quad [-2\sin t\cos^3 t]$

   (b) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$. $\quad \left[y = -\frac{1}{2}x + 2\right]$

6. Let

$$I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x \; dx, \quad n \geq 0$$

(a) Prove that, for $n \geq 2$,

$$I_n = \frac{n}{4}\left(\frac{\pi}{4}\right)^{n-1} - \frac{1}{4}n(n-1)I_{n-2}$$

(b) Find the exact value of I_2.

(c) Show that $I_4 = \frac{1}{64}(\pi^3 - 24\pi + 48)$

7. Given that

$$I = \int_1^5 x^2 \ln x \; dx$$

(i) determine the exact value of I using integration by parts. $\left[\frac{125 \ln 5}{3} - \frac{124}{9}\right]$

(ii) use the trapezium rule with 4 trapezia to obtain an estimate of I. [54.9588]