UNIT 2 TEST 1 SBA 2017

PREVIEW

1. Differentiate with respect to x.

(a) $x\cos^{-1}\left(\frac{x}{2}\right)$ [3]

(b) $\frac{\ln(x^3+2)}{x}$ [3]

(a) $\cos^{-1}\left(\frac{x}{2}\right)-\frac{x}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$ (b) $\frac{3x}{x^3+2}-\frac{\ln(x^3+2)}{x^2}$

2. A curve C has equation $2^x+2y^2=4xy$

Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(6, 4)$. [7]

$$8\ln 2-2=\frac{dy}{dx}$$

3. A curve C has parametric equations

$$x=\cos^2 t\,,\quad y=2\cot t\,,\qquad 0\le t<\frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t. [4]

The tangent to C at the point where $t=\frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P. [6]

(a) $\frac{dy}{dx}=\frac{\csc^2 t}{\sin t\cos t}$ (b) $x=-\frac{1}{8}$

4. Using the substitution $u=\sin x+2$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\sin x+}\ \cos x\ dx=e^2(e-1)$$

[6]

5. (a) Use the trapezium rule with 6 strips, to approximate correct to 3 decimal places,

$$\int_1^4 x^2\ln x\ dx$$

[22.868]

(b) Determine the exact value, in the form $\frac{1}{3}(a \ln 2 - b)$, of

$$\int_1^4 x^2 \ln x \; dx$$

[7]

$\frac{1}{3}(128 \ln 2 - 21)$

6. Given that $z = \sqrt{2} - i$

(a) Determine $\frac{z}{z^*}$ [3]

(b) Find the value of $\left|\frac{z}{z^*}\right|$ [2]

(c) Verify, for $z = \sqrt{2} - i$, that $\arg\left(\frac{z}{z^*}\right) = \arg z - \arg z^*$ [4]

(d) Display on a single Argand diagram z, z^* and $\frac{z}{z^*}$. [3]

(a) $\frac{1}{3} - \frac{2\sqrt{2}}{3}i$ (b) 1

7. Given that $2 + i$ is a root of the equation $f(x) = 0$, where

$$f(x) = 2x^3 + ax^2 + bx - 60 \quad a, b \in \mathbb{R}$$

(a) find the other two roots of the equation $f(x) = 0$, [4]

(b) find the value of a and the value of b. [3]

(a) $2 - i, 6$ (b) $a = -20, b = 58$