1. (a) (i) $\alpha = 1-3i \quad \Rightarrow \beta = 1+3i$

$\alpha + \beta = 1+3i+1-3i = 2$

$\alpha\beta = (1-3i)(1+3i) = 1+9 = 10$

(ii) sum of roots: $\dfrac{1}{\alpha-2}+\dfrac{1}{\beta-2}$

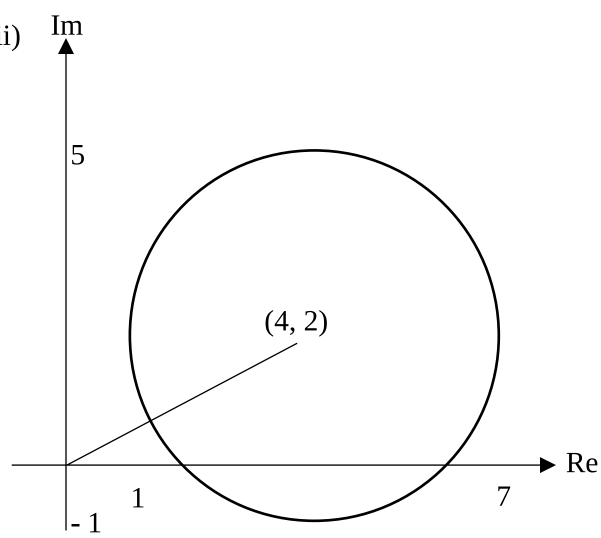
$$\frac{\alpha+\beta-4}{\alpha\beta-2(\alpha+\beta)+4} = \frac{-2}{10} = \frac{-1}{5}$$

product of roots: $\dfrac{1}{(\alpha-2)}\cdot\dfrac{1}{(\beta-2)}$

$$\frac{1}{\alpha\beta-2(\alpha+\beta)+4} = \frac{1}{10-4+4} = \frac{1}{10}$$

equation: $x^2+\dfrac{1}{5}x+\dfrac{1}{10}=0 \quad \Rightarrow 10x^2+2x+1=0$

(b) (i) and (ii)



(iii) $|z| = \dfrac{|u|^5}{|v|^5}$

$|u| = \sqrt{16+4} = 2\sqrt{5}$

$|v| = \sqrt{1+8} = 3$

$$|z| = \left(\frac{2\sqrt{5}}{3}\right)^5 = \frac{800\sqrt{5}}{243} = 7.36$$

$\arg z = \arg u^5 - \arg v^5 = 5\arg u - 5\arg v$

$\arg u = \tan^{-1}\left(\dfrac{1}{2}\right) = 26.57°$

$\arg v = \tan^{-1}\left(2\sqrt{2}\right) = 70.53°$

$\arg z = 5(26.57-70.53) = -219.8° = 140.2°$

(c) $\frac{dx}{dt} = -4\sin t$ $\frac{dy}{dt} = 6\cos 2t$

$$\frac{dy}{dx} = \frac{6\cos 2t}{-4\sin t}$$

$$\frac{dy}{dx} = 0 \Rightarrow \cos 2t = 0$$

$$0 \le 2t \le 2\pi \Rightarrow 2t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$t = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$t = \frac{\pi}{4} \Rightarrow x = 4\cos\frac{\pi}{4} = 2\sqrt{2}$$

$$t = \frac{3\pi}{4} \Rightarrow x = 4\cos\frac{3\pi}{4} = -2\sqrt{2}$$

2. (a)

$$\frac{\partial w}{\partial x} = \frac{2(x-10)-(2x+y)}{(x-10)^2} \times \frac{1}{\frac{2x+y}{x-10}}$$

$$= \frac{2x-20-2x-y}{(x-10)^2} \times \frac{x-10}{2x+y}$$

$$= \frac{-(y+20)}{(x-10)(2x+y)}$$

(b) $\int e^{2x} \sin e^x dx$

let $t = e^x$ [substitution]

$$\frac{dt}{dx} = e^x \Rightarrow dt = e^x dx$$

$$\int e^{2x} \sin e^x dx = \int e^x \sin e^x . e^x dx = \int t \sin t dt$$

let $u = t$ $dv = \sin t$ [by parts]

$du = 1$ $v = -\cos t$

$$\int t \sin t dt = -t\cos t + \int \cos t dt$$

$$= -t\cos t + \sin t + c$$

$$= -e^x \cos e^x + \sin e^x + c$$

(c) (i) $h = \frac{5-2}{3} = 1$

x	2	3	4	5
$f(x)$	$\frac{11}{5}$	$\frac{9}{10}$	$\frac{9}{17}$	$\frac{19}{52}$

$$\text{Area} = \frac{1}{2}\left[\frac{11}{5} + 2\left(\frac{9}{10} + \frac{9}{17}\right) + \frac{19}{52}\right]$$

$$= 2.71$$

(ii) $$\frac{x^2+2x+3}{(x-1)(x^2+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$$

$$x^2+2x+3=A(x^2+1)+(Bx+C)(x-1)$$

compare coeffients

$x^2:\quad 1=A+B\qquad [1]$

$x^1:\quad 2=-B+C\qquad [2]$

$x^0:\quad 3=A-C\qquad [3]$

$[1]+[2]+[3]\Rightarrow 2A=6\Rightarrow A=3$

$[2]+[3]\Rightarrow A-B=5\Rightarrow 3-5=B\Rightarrow -2=B$

$[3]\Rightarrow 3=3-C\Rightarrow 0=C$

$$\therefore \frac{x^2+2x+3}{(x-1)(x^2+1)}=\frac{3}{x-1}-\frac{2x}{x^2+1}$$

(iii) $$\int_2^5 f(x)dx=\int_2^5\frac{3}{x-1}dx-\int_2^5\frac{2x}{x^2-1}dx$$

$$=\left[3\ln|x-1|-\ln|x^2+1|\right]_2^5$$

$$=[3\ln 4-\ln 26]-[3\ln 1-\ln 5]$$

$$=2.51$$

3. (a) $u_{10}=u_8+x(u_9)'$

$$34x+1=13x+1+x(u_9)'$$

$$21x=x(u_9)'$$

$$21=(u_9)'$$

(b) (i) $$S_n=\sum_{r=1}^{n}r(r-1)=\sum_{r=1}^{n}(r^2-r)=\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}r$$

$$=\frac{n}{6}(n+1)(2n+1)-\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)(2n+1)-3n(n+1)}{6}$$

$$=\frac{n(n+1)[2n+1-3]}{6}$$

$$=\frac{n(n+1)(2n-2)}{6}$$

$$=\frac{n(n+1)(n-1)}{3}$$

$$=\frac{n(n^2-1)}{3}$$

(ii) $$\sum_{1}^{20}r(r-1)=\sum_{1}^{9}r(r-1)+\sum_{10}^{20}r(r-1)$$

$$\sum_{10}^{20}r(r-1)=\sum_{1}^{20}r(r-1)-\sum_{1}^{9}r(r-1)$$

$$=\frac{20(20^2-1)}{3}-\frac{9(9^2-1)}{3}$$

$$=20(133)-3(80)=2420$$

(c) (i) $\dfrac{{}^{2r}P_r\,{}^{n}P_r}{(2r)!}=\dfrac{(2r)!}{(2r-r)!}\cdot\dfrac{n!}{(n-r)!}\cdot\dfrac{1}{(2r)!}$

$$=\frac{1}{r!}\cdot\frac{n!}{(n-r)!}$$

$$=\frac{n!}{r!(n-r)!}$$

$$={}^{n}C_r$$

(ii) $(a+b)^n \Rightarrow {}^{n}C_r a^{n-r}b^r$

therefore the term with x^3 is ${}^{5}C_2(3x)^3(2)^2$

$$=1080x^3$$

coefficient: 1080

4. (a) (i) $f(x)=\sqrt[6]{4x^2+4x+1}$

$$=(4x^2+4x+1)^{\frac{1}{6}}$$

$$=\left[(2x+1)^2\right]^{\frac{1}{6}}$$

$$=(1+2x)^{\frac{1}{3}}$$

(ii) $f(x)=1+\dfrac{1}{3}(2x)+\dfrac{\frac{1}{3}\left(\frac{-2}{3}\right)(4x^2)}{2}+\dfrac{\frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)(8x^3)}{6}+\dfrac{\frac{1}{3}\left(\frac{-2}{3}\right)\left(\frac{-5}{3}\right)\left(\frac{-8}{3}\right)(16x^4)}{24}+\dots$

$$=1+\frac{2}{3}x-\frac{4}{9}x^2+\frac{40}{81}x^3-\frac{160}{243}x^4+\dots$$

(iii) $f(0.4)\approx 1+\dfrac{2}{3}(0.4)-\dfrac{4}{9}(0.4)^2+\dfrac{40}{81}(0.4)^3-\dfrac{160}{243}(0.4)^4$

$$\approx 1.22$$

(b) (i) $h(0)=-1$

$h(1)=1$

since *h* is continuous, by the IVT a root exists in the interval

(ii) $x_2=\dfrac{1}{(0.7)^2+1}=0.671$

$x_3=\dfrac{1}{(0.671)^2+1}=0.690$

$x_4=\dfrac{1}{(0.690)^2+1}=0.677$

$x_5=\dfrac{1}{(0.677)^2+1}=0.686$

$x_6=\dfrac{1}{(0.686)^2+1}=0.680$

$x_7=\dfrac{1}{(0.680)^2+1}=0.684$

therefore root is 0.68

(c) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{e^{4x-3} - 4}{4e^{4x-3}}$$

$$= x_n - \frac{1}{4} + \frac{1}{e^{4x-3}}$$

$$x_n - 0.25 + e^{3-4x}$$

$$x_2 = 1 - 0.25 + e^{-1} = 1.1179$$

$$x_3 = 1.1179 - 0.25 + e^{3-4(1.1179)} = 1.0975$$

$$x_4 = 1.0975 - 0.25 + e^{3-4(1.0975)} = 1.0966$$

$$x_5 = 1.0966 - 0.25 + e^{3-4(1.0966)} = 1.0966$$

$$x \approx 1.097$$

5. (a) (i) $^{13}P_8 = 51{,}891{,}840$

(ii) case 1: the friends go on the bus

$$^3C_3 . ^5C_2 = 10$$

case 2: the friends don't go on the bus

$$^3C_0 . ^5C_5 = 1$$

$$10 + 1 = 11$$

(b) $P\,(G \text{ and } A \text{ open}) = \frac{2! \times 3!}{5!}$

$$= 0.1$$

(c) (i) $\det A = 0 + 4\begin{vmatrix} 2 & -1 \\ -1 & 0 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ -1 & 6 \end{vmatrix}$

$$= 4(-1) - 3(13)$$

$$= -43$$

(ii) cofactor of A: $\begin{bmatrix} \begin{vmatrix} 4 & 3 \\ 6 & 0 \end{vmatrix} & -\begin{vmatrix} 0 & 3 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 0 & 4 \\ -1 & 6 \end{vmatrix} \\ -\begin{vmatrix} 1 & -1 \\ 6 & 0 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ -1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ -1 & 6 \end{vmatrix} \\ \begin{vmatrix} 1 & -1 \\ 4 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 0 & 4 \end{vmatrix} \end{bmatrix}$

$$= \begin{bmatrix} -18 & -3 & 4 \\ -6 & -1 & -13 \\ 7 & -6 & 8 \end{bmatrix}$$

Adjoint A: $\begin{bmatrix} -18 & -6 & 7 \\ -3 & -1 & -6 \\ 4 & -13 & 8 \end{bmatrix}$

Therefore $A^{-1} = \frac{1}{-43}\begin{bmatrix} -18 & -6 & 7 \\ -3 & -1 & -6 \\ 4 & -13 & 8 \end{bmatrix}$

6. (a) (i) $2 \times 2 \times 6 = 24$

(ii) P(one head) = P(HT) + P(TH)

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

(iii) P(HTE) + P(THE) + P(HHE)

$$= \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{1}{2}$$

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

(b) $y = C_1 x + C_2 x^2$

$y' = C_1 + 2C_2 x$

$y'' = 2C_2$

$$\frac{x^2}{2} y'' - xy' + y = \frac{x^2}{2}(2C_2) - x(C_1 + 2C_2 x) + (C_1 x + C_2 x^2)$$

$$= C_2 x^2 - C_1 x - 2C_2 x^2 + C_1 x + C_2 x^2$$

$$= C_2 x^2 - 2C_2 x^2 + C_2 x^2 - C_1 x + C_1 x$$

$$= 0$$

therefore $y = C_1 x + C_2 x^2$ is a solution.

(c) (i) $3(x^2 + x)\dfrac{dy}{dx} = 2y(1 + 2x)$

$$\frac{1}{y} dy = \frac{2}{3} \cdot \frac{1 + 2x}{x^2 + x} dx$$

$$\int \frac{1}{y} dy = \frac{2}{3} \int \frac{1 + 2x}{x^2 + x} dx$$

$$\ln|y| = \frac{2}{3} \ln|x^2 + x| + k$$

$$\ln|y| = \ln|x^2 + x|^{\frac{2}{3}} + \ln C$$

$$\ln|y| = \ln C \sqrt[3]{(x^2 + x)^2}$$

$$y = C \sqrt[3]{(x^2 + x)^2}$$

(ii) $y = 1, x = 1$

$$1 = C \sqrt[3]{(1^2 + 1)^2}$$

$$1 = C \sqrt[3]{4}$$

$$\frac{1}{\sqrt[3]{4}} = C$$

$$\therefore y = \frac{1}{\sqrt[3]{4}} \cdot \sqrt[3]{(x^2 + x)^2} = \sqrt[3]{\frac{(x^2 + x)^2}{4}}$$