UNIT 1 – TEST 2 (PREVIEW) 2015

1 i) $\dfrac{\cos 2\theta}{\sin 2\theta} + \dfrac{1}{\sin 2\theta}$

$= \dfrac{\cos 2\theta + 1}{\sin 2\theta}$

$= \dfrac{2\cos^2\theta - 1 + 1}{2\sin\theta\cos\theta}$

$= \dfrac{\cos^2\theta}{\sin\theta\cos\theta} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

(ii) using the result in (i)

$\cot 15^\circ = \dfrac{\cos 30}{\sin 30} + \dfrac{1}{\sin 30}$

$= \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} + \dfrac{1}{\frac{1}{2}}$

$= \sqrt{3} + 2$

2 (i) $R\sin(\theta - \alpha) = R(\sin\theta\cos\alpha - \sin\alpha\cos\theta)$

$\therefore$ If $4\sin\theta - 3\cos\theta = R\cos\alpha\sin\theta - R\sin\alpha\cos\theta$

$R\cos\alpha = 4 \quad R\sin\alpha = 3$

$R = \sqrt{4^2 + 3^2} = 5$

$\tan\alpha = \dfrac{3}{4} \Rightarrow \alpha = \tan^{-1}\dfrac{3}{4} = 36.9^\circ$

So $4\sin\theta - 3\cos\theta = 5\sin(\theta - 36.9^\circ)$

(ii) $5\sin(\theta - 36.9) + 1 = 0 \qquad -180 \leq \theta \leq 180$

$\sin(\theta - 36.9) = -\frac{1}{5}$

so $(\theta - 36.9)$ is in 3rd or 4th quadrants

$\sin^{-1}\left(\frac{1}{5}\right) = 11.5^\circ$

so $\theta - 36.9^\circ = -180 + 11.5 = -168.5$

or $= -11.5 \qquad = -11.5$

$\theta = -131.6, \quad 25.4^\circ$

(b) $5k + c = 43$

$-5k + c = -37$

$10k = 80 \Rightarrow k = 8$

$c = 3$

3. $x^2 + y^2 - 12x - 8y + 44 = 0$

$x^2 - 12x + y^2 - 8y + 44 = 0$

$x^2 - 12x + (-6)^2 + y^2 - 8y + (-4)^2 + 44 - (-6)^2 - (-4)^2 = 0$

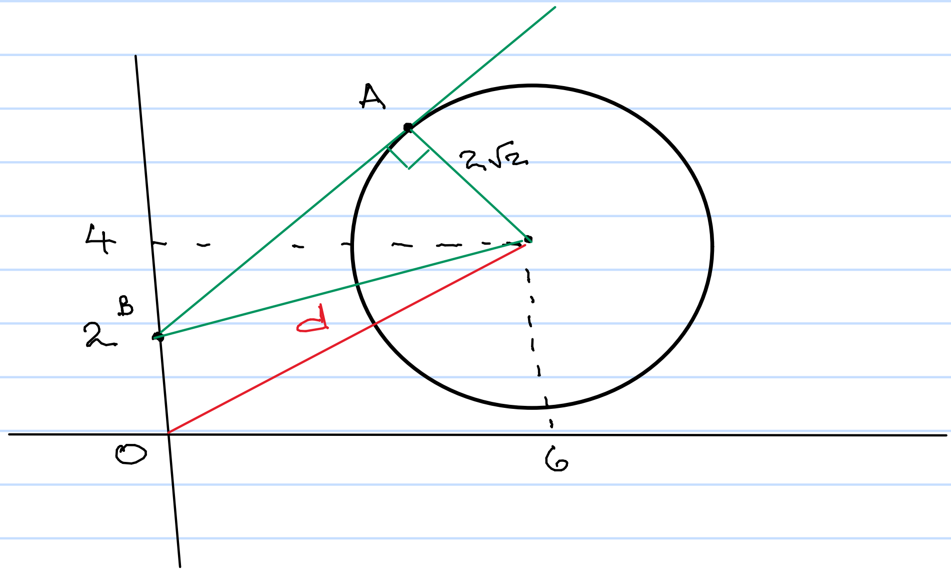
$(x-6)^2 + (y-4)^2 + 44 - 36 - 16 = 0$

$(x-6)^2 + (y-4)^2 - 8 = 0$

so centre $(6, 4)$ radius $= \sqrt{8}$

$= 2\sqrt{2}$

(3)



(b) $d^2 = 6^2 + 4^2 = 36 + 16 = 52$

$d = \sqrt{52} = 2\sqrt{13}$

(c) distance of B to centre $= \sqrt{6^2 + 2^2} = \sqrt{40}$

so $AB = \sqrt{(\sqrt{40})^2 - (2\sqrt{2})^2} = \sqrt{40 - 8} = \sqrt{32}$

$= \sqrt{16 \times 2}$

$= 4\sqrt{2}$

4) $16x^2 + y^2 = 64$

when $x = 0$ $y = \pm 8$

and when $y = 0$ $x = \pm 2$

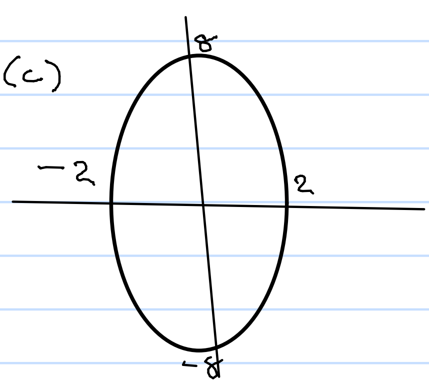
So y - intercepts are $(0, 8)$ and $(0, -8)$

and x - intercepts are $(2, 0)$ and $(-2, 0)$

(b) Major axis $= 16$

minor axis $= 4$

(c)



5. (i) $\overrightarrow{PR} = 4\underset{\sim}{i} + 4\underline{j} + 4\underset{\sim}{k}$

$\overline{PQ} = -4i + 8k + 4j$

$= -4\underset{\sim}{i} + 4\underset{\sim}{j} + 8\underset{\sim}{k}$

$\overline{PQ} \cdot \overline{PR} = -16 + 16 + 32 = 32$

$= \sqrt{48} \cdot \sqrt{96} \cos(Q\hat{P}R)$

$\cos(Q\hat{P}R) = \dfrac{32}{\sqrt{48}\sqrt{96}}$

$Q\hat{P}R = \cos^{-1}(0.4714) = 61.8^{0}$

$\overline{QR} = -8k + 8i + 4k$

$= -4k + 8i$

$|QR| = \sqrt{80}$

Perimeter $= \sqrt{80} + \sqrt{48} + \sqrt{96}$

$= 25.7$ units

6. $\overline{OC} = 3\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix}$

$\overline{OD} = 2\begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 8 \\ 0 \\ 12 \end{pmatrix}$

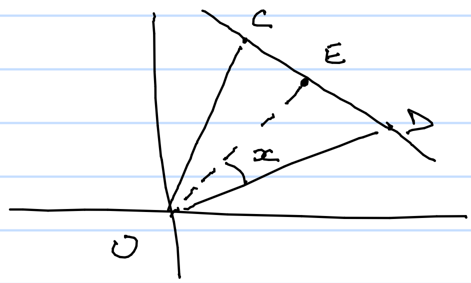
$\overline{CD} = \begin{pmatrix} 8 \\ 0 \\ 12 \end{pmatrix} - \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

$|CD| = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$

unit vector in the direction of CD

$= \dfrac{1}{7}\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

6 (ii)



$$\vec{CE} = \frac{1}{2}\begin{pmatrix}2\\-3\\6\end{pmatrix}$$

$$\vec{OE} = \begin{pmatrix}6\\3\\6\end{pmatrix} + \frac{1}{2}\begin{pmatrix}2\\-3\\6\end{pmatrix} = \begin{pmatrix}7\\\frac{3}{2}\\9\end{pmatrix}$$

$$\overline{OE}\cdot\overline{OD} = \begin{pmatrix}7\\\frac{3}{2}\\9\end{pmatrix}\cdot\begin{pmatrix}8\\0\\12\end{pmatrix} = 56 + 108 = 164$$

$$= \sqrt{132.25}\cdot\sqrt{208}\ \cos x$$

$$\cos x = \frac{164}{\sqrt{132.25}\ \sqrt{208}}$$

$$x = \cos^{-1} 0.9888 = 8.6^\circ$$

7 (i) $\overline{OP} = \begin{pmatrix}3\\1\\2\end{pmatrix}$ $\vec{OQ} = \begin{pmatrix}0\\-1\\4\end{pmatrix}$

$$\overline{PQ} = \begin{pmatrix}0\\-1\\4\end{pmatrix} - \begin{pmatrix}3\\1\\2\end{pmatrix} = \begin{pmatrix}-3\\-2\\2\end{pmatrix}$$

equation of line passing through P and Q

$$r = \begin{pmatrix}3\\1\\2\end{pmatrix} + t\begin{pmatrix}-3\\-2\\2\end{pmatrix}$$

$$= (3i + j + 2k) + t(-3i - 2j + 2k)$$

7(ii) $\overline{OS} = \begin{pmatrix} 9 \\ 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$

for $t = -2$ shows S is on line.

$\overline{PS} = \begin{pmatrix} 9 \\ 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ -4 \end{pmatrix}$

$\overline{QS} = \begin{pmatrix} 9 \\ 5 \\ -2 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} 9 \\ 6 \\ -6 \end{pmatrix}$

$\overline{QS} = \frac{3}{2} \overline{PS}$

(iii) Line L had direction vector

$$\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$$

other line has direction vector

$$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

$\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} = -3 - 8 + 4 = -7$

$= \sqrt{17} \cdot \sqrt{21} \cos \alpha$

$\cos \alpha = \dfrac{-7}{\sqrt{17}\sqrt{21}} = 111.7^\circ$

but the acute angle $180 - 111.7 = 68.3^\circ$