1 (a) Given that

$$\frac{3+\sin^2\theta}{\cos\theta-2} = 3\cos\theta$$

show that

$$\cos\theta = -\tfrac{1}{2}$$ *(4)*

(b) Hence solve the equation

$$\frac{3+\sin^2 3x}{\cos 3x-2} = 3\cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4)*

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2. $$f(x) = 12\cos x - 4\sin x.$$

Given that $f(x) = R\cos(x+\alpha)$, where $R \geqslant 0$ and $0 \leqslant \alpha \leqslant 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12\cos x - 4\sin x = 7$$

for $0 \leqslant x < 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12\cos x - 4\sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)

3. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

Find a cartesian equation of the curve in the form $y^2 = \mathrm{f}(x)$.

(4)

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4. The line l_1 has vector equation

$$\mathbf{r} = 8\mathbf{i} + 12\mathbf{j} + 14\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k}),$$

where λ is a parameter.

The point A has coordinates (4, 8, a), where a is a constant. The point B has coordinates (b, 13, 13), where b is a constant. Points A and B lie on the line l_1.

(a) Find the values of a and b.

(3)

Given that the point O is the origin, and that the point P lies on l_1 such that OP is perpendicular to l_1,

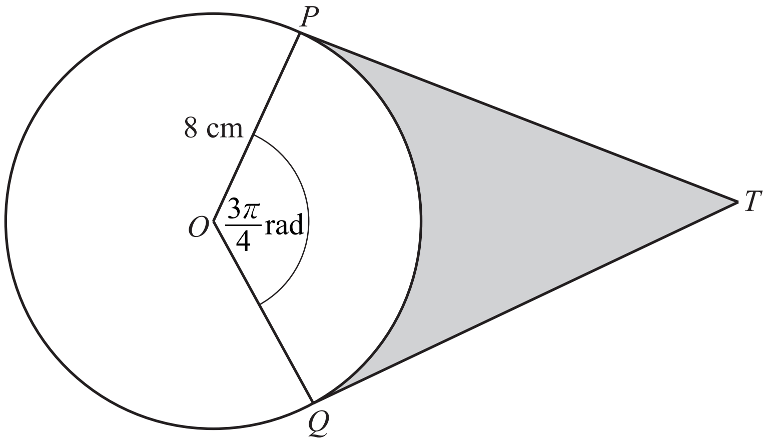
(b) find the coordinates of P.

(5)

(c) Hence find the distance OP, giving your answer as a simplified surd.

(2)

5



The diagram shows a circle, centre O, radius 8 cm. The points P and Q lie on the circle. The lines PT and QT are tangents to the circle and angle $POQ = \frac{3\pi}{4}$ radians.

(i) Find the length of PT. [2]

(ii) Find the area of the shaded region. [3]

(iii) Find the perimeter of the shaded region. [2]

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6. The points A and B have coordinates (–2, 11) and (8, 1) respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates (3, 6),

(1)

(b) find an equation for C,

(4)

(c) Verify that the point (10, 7) lies on C.

(1)

(d) Find an equation of the tangent to C at the point (10, 7), giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

Q	Solution	Marks	Total	Comments
1(a)	$\frac{3+\sin^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \frac{3+(1-\cos^2\theta)}{\cos\theta-2}=3\cos\theta$	M1		$\cos^2\theta + \sin^2\theta = 1$ stated or used [If cand starts with cos $\theta = -½$ and gets $\sin^2\theta = ¾$ without explicitly finding value for θ and verifies 1st equation is true, award M1moA0]
	$\Rightarrow \frac{4-\cos^2\theta}{\cos\theta-2}=3\cos\theta$			
	$\Rightarrow \frac{(2-\cos\theta)(2+\cos\theta)}{\cos\theta-2}=3\cos\theta$	m1		Difference of two squares or division (PI by next line)
	$\Rightarrow -1(2+\cos\theta)=3\cos\theta$	A1		
	$\Rightarrow -2=4\cos\theta \Rightarrow \cos\theta=-\frac{1}{2}$	A1	4	CSO AG
	Alternative for (a)			
	$3+1-\cos^2\theta=3\cos^2\theta-6\cos\theta$	(M1)		$\cos^2\theta + \sin^2\theta = 1$
	$(4\cos\theta+2)(\cos\theta-2)=0$	(m1)		Factorising or formula
	$\cos\theta - 2 \neq 0$	(A1)		Indicates rejection of $\cos\theta=2$
	$\Rightarrow 4\cos\theta=-2 \Rightarrow \cos\theta=-\frac{1}{2}$	(A1)		AG Be convinced
(b)	$\theta = 3x \Rightarrow \cos 3x = -\frac{1}{2}$	M1		Uses part (a) to reach either cos $3x = -0.5$ or cos$3x = 0.5$
	$\cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$	m1		Or $\cos^{-1}(0.5) = 60^\circ$ Condone radians here
	$3x = 120^\circ, 240^\circ, 480^\circ, \ldots$			
	$x = 40^\circ, 80^\circ, 160^\circ$	A2,1,0	4	A1 for at least <u>two</u> correct. If >3 solutions in the interval $0^\circ < x < 180^\circ$, deduct 1 mark from any A marks for each extra solution. Deduct 1 mark from any A marks if answers in radians. Ignore extra values outside the given interval.
	Total		**8**	

2.	(a) $R\cos\alpha = 12, \quad R\sin\alpha = 4$	
	$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$ Accept if just written down, awrt 12.6	M1 A1
	$\tan\alpha = \frac{4}{12}, \quad \Rightarrow \quad \alpha \approx 18.43°$ awrt 18.4°	M1, A1**(4)**
	(b) $\cos(x + \text{ their } \alpha) = \frac{7}{\text{their } R} \quad (\approx 0.5534)$	M1
	$x + \text{their } \alpha = 56.4°$ awrt 56°	A1
	$= \ \ldots\ , 303.6°$ $360° -$ their principal value	M1
	$x = 38.0°, 285.2°$ Ignore solutions out of range	A1, A1 **(5)**
	If answers given to more than 1 dp, penalise first time then accept awrt above.	
	(c)(i) minimum value is $-\sqrt{160}$ ft their R	B1ft
	(ii) $\cos(x + \text{ their } \alpha) = -1$	M1
	$x \approx 161.57°$ cao	A1 **(3)** **[12]**

3.	$x = \tan^2 t = \frac{\sin^2 t}{\cos^2 t} \qquad y = \sin t$		
Way 1			
	$x = \frac{\sin^2 t}{1 - \sin^2 t}$	Uses $\cos^2 t = 1 - \sin^2 t$	M1
	$x = \frac{y^2}{1 - y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	M1
	$x(1 - y^2) = y^2 \ \Rightarrow\ x - xy^2 = y^2$		
	$x = y^2 + xy^2 \ \Rightarrow\ x = y^2(1 + x)$	Rearranging and factorising with an attempt to make y^2 the subject.	ddM1
	$y^2 = \frac{x}{1 + x}$	$\frac{x}{1+x}$	A1
Aliter 3 Way 2	$1 + \cot^2 t = co\sec^2 t$	Uses $1 + \cot^2 t = co\sec^2 t$	M1
	$= \frac{1}{\sin^2 t}$	$\cos ec^2 t = \frac{1}{\sin^2 t}$	M1 implied
	Hence, $1 + \frac{1}{x} = \frac{1}{y^2}$	Eliminates '*t*' *to* write an equation involving *x* and *y*.	ddM1
	Hence, $y^2 = 1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	$1 - \frac{1}{(1+x)}$ or $\frac{x}{1+x}$	A1

4. (a)	$\lambda=-4 \rightarrow a=18, \qquad \mu=1 \rightarrow b=9$	M1 A1, A1 (3)
(b)	$\begin{pmatrix} 8+\lambda \\ 12+\lambda \\ 14-\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 0$	M1
	$\therefore 8+\lambda+12+\lambda-14+\lambda=0$	A1
	Solves to obtain λ ($\lambda=-2$)	dM1
	Then substitutes value for λ to give P at the point (6, 10, 16) (any form)	M1, A1 (5)
(c)	OP = $\sqrt{36+100+256}$	M1
	(= $\sqrt{392}$) $= 14\sqrt{2}$	A1 cao (2) **[10]**

5 (i)	$\frac{PT}{8}=\tan\left(\frac{3\pi}{8}\right)$ oe	M1	$\frac{PT}{\sin\frac{3\pi}{8}}=\frac{8}{\sin\frac{\pi}{8}}$	
	$PT=19.3$	A1	awrt 19.3	
(ii)	$\frac{1}{2}\times 8^2\times\frac{3\pi}{4}$ oe (75.4)	M1	or $\frac{1}{2}\times 8^2\times\frac{3\pi}{8}$	
	$8\tan\left(\frac{3\pi}{8}\right)\times 8-$ *their* sector oe (=154.5-‘75.4’)	M1	$8\times$ *their PT* $-$ *their* sector	
	79.1	A1	awrt 79.1	
(iii)	$8\left(\frac{3\pi}{4}\right)$ oe (18.8)	M1		
	$\left[6\pi+16\tan\left(\frac{3\pi}{8}\right)\right]=57.5$	A1	Accept 57.4 to 57.5	

Question Number	Scheme		Marks
6.			
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50$ $\left(\text{or } (\sqrt{50})^2 \text{ or } (5\sqrt{2})^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive <u>value</u>.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$, } <u>$(10-3)^2 + (7-6)^2 = 50$</u>, {so the point lies on C.}		<u>B1</u>
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x - 10)$	$y - 7 = (\text{their gradient})(x - 10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			[10]

	Notes
(a)	Alternative method: $C\left(-2 + \frac{8--2}{2}, 11 + \frac{1-11}{2}\right)$ or $C\left(8 + \frac{-2-8}{2}, 1 + \frac{11-1}{2}\right)$
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2 + (11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2 + (11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2 + (11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.
(c)	B1 awarded for correct verification of <u>$(10-3)^2 + (7-6)^2 = 50$</u> with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.