TEST CODE **02234020**

FORM TP 2014243

MAY/JUNE 2014

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

PURE MATHEMATICS

UNIT 2 – Paper 02

ANALYSIS, MATRICES AND COMPLEX NUMBERS

2 hours 30 minutes

28 MAY 2014 (p.m.)

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This examination paper consists of THREE sections.
2. Answer ALL questions from the THREE sections.
3. Each section consists of TWO questions.
4. Write your solutions, with full working, in the answer booklet provided.
5. Unless otherwise stated in the question, any numerical answer that is not exact MUST be written correct to three significant figures.

Examination Materials Permitted

Graph paper (provided)
Mathematical formulae and tables (provided) – **Revised 2012**
Mathematical instruments
Silent, non-programmable, electronic calculator

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

SECTION A

Module 1

Answer BOTH questions.

1. (a) (i) Differentiate, with respect to x,

$$y = \ln(x^2 + 4) - x\tan^{-1}\left(\frac{x}{2}\right).$$

[5 marks]

(ii) A curve is defined parametrically as

$$x = a\cos^3 t,\ y = a\sin^3 t.$$

Show that the tangent at the point $P(x, y)$ is the line

$$y\cos t + x\sin t = a\sin t\cos t.$$

[7 marks]

(b) Let the roots of the quadratic equation $x^2 + 3x + 9 = 0$ be α and β.

(i) Determine the nature of the roots of the equation. **[2 marks]**

(ii) Express α and β in the form $re^{i\theta}$, where r is the modulus and θ is the argument, where $-\pi < \theta \leq \pi$. **[4 marks]**

(iii) Using de Moivre's theorem, or otherwise, compute $\alpha^3 + \beta^3$. **[4 marks]**

(iv) Hence, or otherwise, obtain the quadratic equation whose roots are α^3 and β^3. **[3 marks]**

Total 25 marks

2. (a) Let $F_n(x) = \int (\ln x)^n \, dx$.

(i) Show that $F_n(x) = x(\ln x)^n - n F_{n-1}(x)$. **[3 marks]**

(ii) Hence, or otherwise, show that

$$F_3(2) - F_3(1) = 2(\ln 2)^3 - 6(\ln 2)^2 + 12 \ln 2 - 6.$$ **[7 marks]**

(b) (i) By decomposing $\dfrac{y^2 + 2y + 1}{y^4 + 2y^2 + 1}$ into partial fractions, show that

$$\frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1} = \frac{1}{y^2 + 1} + \frac{2y}{(y^2 + 1)^2}.$$ **[7 marks]**

(ii) Hence, find $\displaystyle\int_0^1 \frac{y^2 + 2y + 1}{y^4 + 2y^2 + 1} \, dy$. **[8 marks]**

Total 25 marks

SECTION B

Module 2

Answer BOTH questions.

3. (a) (i) Prove, by mathematical induction, that for $n \in \mathrm{N}$

$$S_n = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots\ldots + \frac{1}{2^{n-1}} = 2 - \frac{1}{2^{n-1}}.$$ **[8 marks]**

(ii) Hence, or otherwise, find $\lim_{n \to \infty} S_n$. **[3 marks]**

(b) Find the Maclaurin expansion for

$f(x) = (1 + x)^2 \sin x$

up to and including the term in x^3. **[14 marks]**

Total 25 marks

4. (a) (i) For the binomial expansion of $(2x + 3)^{20}$, show that the ratio of the term in x^6 to the term in x^7 is $\frac{3}{4x}$. **[5 marks]**

(ii) a) Determine the FIRST THREE terms of the binomial expansion of $(1 + 2x)^{10}$.

b) Hence, obtain an estimate for $(1.01)^{10}$. **[7 marks]**

(b) Show that $\frac{n!}{(n-r)!r!} + \frac{n!}{(n-r+1)!(r-1)!} = \frac{(n+1)!}{(n-r+1)!r!}$. **[6 marks]**

(c) (i) Show that the function $f(x) = -x^3 + 3x + 4$ has a root in the interval $[1, 3]$. **[3 marks]**

(ii) By taking $x_1 = 2.1$ as a first approximation of the root in the interval $[1, 3]$, use the Newton–Raphson method to obtain a **second** approximation, x_2, in the interval $[1, 3]$. **[4 marks]**

Total 25 marks

SECTION C

Module 3

Answer BOTH questions.

5. (a) (i) Five teams are to meet at a round table. Each team consists of two members AND one leader. How many seating arrangements are possible if each team sits together with the leader of the team in the middle? **[7 marks]**

(ii) In an experiment, individuals were asked to colour a shape by selecting from two available colours, red and blue. The individuals chose one colour, two colours or no colour.

In total, 80% of the individuals used colours and 600 individuals used no colour.

a) Given that 40% of the individuals used red and 50% used blue, calculate the probability that an individual used BOTH colours. **[4 marks]**

b) Determine the TOTAL number of individuals that participated in the experiment. **[2 marks]**

(b) **A** and **B** are the two matrices given below.

$$\mathbf{A} = \begin{pmatrix} 1 & x & -1 \\ 3 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 2 & 5 \\ 2 & 3 & 4 \\ 1 & 1 & 2 \end{pmatrix}$$

(i) Determine the range of values of x for which $\mathbf{A}^{-1}$ exists. **[4 marks]**

(ii) Given that det $(\mathbf{AB}) = -21$, show that $x = 3$. **[4 marks]**

(iii) Hence, obtain $\mathbf{A}^{-1}$. **[4 marks]**

Total 25 marks

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6. (a) (i) Show that the general solution of the differential equation

$$y' + y \tan x = \sec x$$

is $\quad y = \sin x + C \cos x.$ **[10 marks]**

(ii) Hence, obtain the particular solution where $y = \frac{2}{\sqrt{2}}$ and $x = \frac{\pi}{4}$. **[4 marks]**

(b) A differential equation is given as $y'' - 5y' = xe^{5x}$. Given that a particular solution is $y_p(x) = Ax^2 e^{5x} + Bxe^{5x}$, solve the differential equation. **[11 marks]**

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.