Solutions to SBA Unit 2 Test 2 (2014)

1.

a) $x_2 = 1(p+1) = p+1$ (1) [1]

b) $x_3 = (p+1)(p+p+1)$ (1)

$= (p+1)(2p+1)$

$= 2p^2 + 3p + 1$ as required (1) [2]

c) $2p^2 + 3p + 1 = 1$ (1)

$2p^2 + 3p = 0$ (1)

$p(2p+3) = 0$

$p = 0 \text{ or } p = -\frac{3}{2}$

$\therefore p = -\frac{3}{2}$ (1) [3]

d) $x_1 = 1,\ x_2 = -\frac{1}{2}$,

$x_3 = 1,\ x_4 = -\frac{1}{2}$

$\therefore\ x_{2008} = -\frac{1}{2}$ (1) [1]

2. a) $ar = 192$ (1)

$ar^2 = 144$ (1)

$\frac{r^2}{r} = \frac{144}{192}$ (1)

$r = \frac{3}{4}$ (1) [4]

b) $\frac{3}{4}a = 192$ (1)

$a = 256$ (1) [2]

c) $S = \frac{a}{1-r}$

$= \frac{256}{1-\frac{3}{4}}$ (1)

$= 1024$ (1) [2]

d) $S_n = \frac{a(1-r^n)}{1-r} > 1000$

$\frac{256\left(1-\left(\frac{3}{4}\right)^n\right)}{1-\frac{3}{4}} > 1000$ (1)

$256\left(1-\left(\frac{3}{4}\right)^n\right) > 250$

$1-\left(\frac{3}{4}\right)^n > \frac{125}{128}$ (1)

$\left(\frac{3}{4}\right)^n < \frac{3}{128}$ (1)

$n\log\left(\frac{3}{4}\right) < \log\left(\frac{3}{128}\right)$ (1)

$n > 13.05$ (1)

$n = 14$ (1) [6]

3. a) $(1+4x^2)^{\frac{1}{2}}$

$= 1 + \frac{1}{2}(4x^2) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(4x^2)^2$ (1) + (1)

$= 1 + 2x^2 - 2x^4$ (1) [3]

b) $-1 < 4x^2 < 1$ (1)

$-\frac{1}{2} < x < \frac{1}{2}$ (1) [2]

c) $\int_0^{\frac{1}{4}} (1+4x^2)^{\frac{1}{2}}\,dx$

$= \int_0^{\frac{1}{4}} 1 + 2x^2 - 2x^4\,dx$ (1)

$= \left[x + \frac{2}{3}x^3 - \frac{2}{5}x^5\right]_0^{\frac{1}{4}}$ (1)

$= \frac{1}{4} + \frac{2}{3}\left(\frac{1}{4}\right)^3 - \frac{2}{5}\left(\frac{1}{4}\right)^5$

$= 0.260$ (1) [3]

4. a) $f(r)=(r-1)r(r+1)(r+2)$

$f(r+1)=r(r+1)(r+2)(r+3)$ (1)

$f(r+1)-f(r)$

$=r(r+1)(r+2)(r+3)-(r-1)r(r+1)(r+2)$

$=r(r+1)(r+2)(r+3-r+1)$ (1)

$=4r(r+1)(r+2)$ as required (1) [3]

$\therefore k=4$

b) $\sum_{r=1}^{n} r(r+1)(r+2)$

$$=\frac{1}{4}\left[\sum_{1}^{n} f(r+1)-f(r)\right]$$ (1)

$$=\frac{1}{4}\left[\begin{array}{c} \cancel{24}-0 \\ \cancel{120}-\cancel{24} \\ \cancel{360}-\cancel{120} \\ \vdots \\ \cancel{(n-1)n(n+1)(n+2)}-\cancel{(n-2)(n-1)n(n+1)} \\ n(n+1)(n+2)(n+3)-\cancel{(n-1)n(n+1)(n+2)} \end{array}\right]$$ (1) + (1)

$$=\frac{1}{4}n(n+1)(n+2)(n+3)$$ (1) [4]

c) Let P(n) be $\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$

When $n=1$

LHS $=\frac{2}{(1)(2)(3)}=\frac{1}{3}$ (1)

RHS $=\frac{1}{2}-\frac{1}{(2)(3)}=\frac{1}{3}$ (1)

Therefore LHS=RHS so $P(n)$ is true for $n=1$ (1)

Assume $P(n)$ is true for $n=k$

RHS $=\frac{1}{2}-\frac{1}{(k+1)(k+2)}$ (1)

When $n=k+1$

RHS $=\frac{1}{2}-\frac{1}{(k+2)(k+3)}$ (1)

LHS $=\frac{1}{2}-\frac{1}{(k+1)(k+2)}+\frac{2}{(k+1)(k+2)(k+3)}$ (1)

$=\frac{1}{2}-\frac{k+3-2}{(k+1)(k+2)(k+3)}$ (1)

$=\frac{1}{2}-\frac{\cancel{k+1}}{\cancel{(k+1)}(k+2)(k+3)}$

$=\frac{1}{2}-\frac{1}{(k+2)(k+3)}$ (1)

Therefore LHS=RHS so $P(n)$ is true for $n=k+1$ (1)

So by Mathematical Induction $\sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)}$

$=\frac{1}{2}-\frac{1}{(n+1)(n+2)}$ is true for all positive integers n (1)

[10]

d) $\sum_{r=1}^{\infty}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}$

As $n\to\infty$ $\frac{1}{(n+1)(n+2)}\to 0$ (1)

$\therefore \sum_{r=1}^{\infty}\frac{2}{r(r+1)(r+2)}=\frac{1}{2}$ (1) [2]

5. a) $\theta - 20 = 0$

$t + 26 - 20e^{-0.5t} - 20 = 0$

$t - 20e^{-0.5t} + 6 = 0$ } (1)

$f(t)=t-20e^{-0.5t}+6$

$a=1.8$ $f(1.8)=-0.3314$ (1)

$b=2$ $f(2)=0.6424$ (1)

$$x=\frac{af(b)-bf(a)}{f(b)-f(a)}$$

$$=\frac{1.8(0.6424)-2(-0.3314)}{0.6424-(-0.3314)}$$ (1)

$=1.87$ (1) [5]

$b)\ f(t) = t - 20e^{-0.5t} + 6$

$f'(t) = 1 + 10e^{-0.5t}$ (1) + (1)

$f(1.9) = 0.16518$ (1)

$f'(1.9) = 4.86741$ (1)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_2 = 1.9 - \frac{0.16518}{4.86741}$ (1)

$= 1.866$ (1) [6]

c) $t = 1.866 \times 60$

$= 112\ mins$ (1) [1]