Solutions to SBA Unit 2 Test 3 (2015)

1. (a)(i) Number of ways to choose 6 books
$= {}^{18}C_6$ (1)
$= 18564$ (1) [2]

(ii) Number of choices with HP book
$= {}^{17}C_5$ (1)
$= 6188$ (1) [2]

(b) (i) Number of ways $= 8!$ (1)
$= 40320$ (1) [2]

(ii)Number of ways with boys next to each other
$= 5! \times 4!$ (1)
$= 2880$ (1) [2]

2. (i) P(both toffees)
$= \left(\frac{1}{3} \times \frac{6}{10} \times \frac{5}{9}\right)$ (1) $+ \left(\frac{1}{3} \times \frac{5}{8} \times \frac{4}{7}\right)$ (1)
$+ \left(\frac{1}{3} \times \frac{3}{10} \times \frac{2}{9}\right)$ (1)

$= \frac{1}{9} + \frac{5}{42} + \frac{1}{45}$

$= \frac{53}{210}$ (1) [4]

(ii) P(both came from A|both toffees)

$= \frac{\text{P(both came from A} \cap \text{both toffees)}}{\text{both toffees}}$

$= \frac{\frac{1}{3} \times \frac{6}{10} \times \frac{5}{9}\,(1)}{\frac{53}{210}\,(1)}$

$= \frac{70}{159}$ (1) [3]

3. (a) $x\frac{dy}{dx} + 2y = 4x^2$

$\frac{dy}{dx} + \frac{2}{x}y = 4x$ (1)

$I = e^{\int \frac{2}{x}\,dx}$

$= e^{2\ln x}$ (1)

$= x^2$ (1)

$\int \left(x^2\frac{dy}{dx} + 2xy\right)\,dx = \int 4x^3\,\,dx$ (1)

$x^2y = x^4 + c$ (1) [5]

(b) $x^2y = x^4 + c$

$5 = 1 + c$

$c = 4$ (1)

$y = x^2 + \frac{4}{x^2}$ (1) [2]

4. (a) $y = xv$

$\frac{dy}{dx} = x\frac{dv}{dx} + v$ (1)

$\frac{d^2y}{dx^2} = x\frac{d^2v}{dx^2} + \frac{dv}{dx} + \frac{dv}{dx} = x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}$ (1)

$4x^2\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + (8 + 4x^2)y = x^4$

$4x^2\left(x\frac{d^2v}{dx^2} + 2\frac{dv}{dx}\right) - 8x\left(x\frac{dv}{dx} + v\right) + (8 + 4x^2)xv$

$= x^4$ (1)

$4x^3\frac{d^2v}{dx^2} + 8x^2\frac{dv}{dx} - 8x^2\frac{dv}{dx} - 8xv + 8xv + 4x^3v = x^4$ (1)

$4x^3\frac{d^2v}{dx^2} + 4x^3v = x^4$ (1)

$4\frac{d^2v}{dx^2} + 4v = x$ as required (1) [6]

$(b)\ m^2 + 4 = 0$(1)
$m = \pm 2i$(1)

C.F. is $v = A\cos 2x + B\sin 2x$(1)

let P.I be $v = ax + b$
$\frac{dv}{dx} = a$(1)

$\frac{d^2v}{dx^2} = 0$(1)

$4\frac{d^2v}{dx^2} + 4v = x$

$4ax + 4b = x$(1)
$4a = 1$
$a = \frac{1}{4}$(1)
$4b = 0$
$b = 0$(1)
$P.I.\ is\ v = \frac{1}{4}x$

G.S. is $v = Acos\ 2x + Bsin\ 2x + \frac{1}{4}x$(1) [9]

$(c) v = Acos\ 2x + Bsin\ 2x + \frac{1}{4}x$
$\frac{y}{x} = Acos\ 2x + Bsin\ 2x + \frac{1}{4}x$(1)
$y = Axcos\ 2x + Bxsin\ 2x + \frac{1}{4}x^2$(1) [2]

5. (a) $AB = I$

$$\begin{pmatrix}2 & 0 & 7\\ 0 & 1 & 0\\ 1 & -2 & 1\end{pmatrix}\begin{pmatrix}-x & 14x & 7x\\ 0 & 1 & 0\\ x & -4x & -2x\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$ (1)

$$\begin{pmatrix}5x & 0 & 0\\ 0 & 1 & 0\\ 0 & 10x-2 & 5x\end{pmatrix}$$ (1) $= \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$

$5x = 1$
$x = \frac{1}{5}$ (1) [3]

(b) $\begin{vmatrix}2x & 4 & 1\\ 2 & 3 & -1\\ 0 & -2 & x\end{vmatrix} = 6x^2 - 10$

$-(-2)\begin{vmatrix}2x & 1\\ 2 & -1\end{vmatrix} + x\begin{vmatrix}2x & 4\\ 2 & 3\end{vmatrix} = 6x^2 - 10$ (1)

$2(-2x - 2) + x(6x - 8) = 6x^2 - 10$ (1)

$-4x - 4 + 6x^2 - 8x - 6x^2 + 10 = 0$ (1)
$-12x + 6 = 0$ (1)

$x = \frac{1}{2}$ (1) [5]

6. (i) $x + y + z = 120$ (1)
$6x + 4y + 3z = 610$ (1)
$x = 2(y + z)$ (1) [3]

(ii) $\left(\begin{array}{ccc|c}1 & 1 & 1 & 120\\ 6 & 4 & 3 & 610\\ 1 & -2 & -2 & 0\end{array}\right)$ (1) [1]

(iii) $\left(\begin{array}{ccc|c}1 & 1 & 1 & 120\\ 0 & 2 & 3 & 110\\ 0 & 3 & 3 & 120\end{array}\right)$ $6R_1 - R_2$ (1)
$R_1 - R_3$ (1)

$\left(\begin{array}{ccc|c}1 & 1 & 1 & 120\\ 0 & 2 & 3 & 110\\ 0 & 0 & 3 & 90\end{array}\right)$ $3R_2 - 2R_3$(1) [3]

(iv) $3z = 90$
$z = 30$(1)

$2y + 3z = 110$
$2y + 90 = 110$
$y = 10$(1)

$x + y + z = 120$
$x + 10 + 30 = 120$
$x = 80$(1)

In each bouquet 16 roses(1), 2 tulips(1) and 6 lilies(1) [6]