Solutions to SBA Unit 2 Test 2 (2017)

Question 1

a) $n^{th}\ term = (4n+1)$ (1) [1]

b) $5^2 + 9^2 + 13^2 + 17^2 + \cdots$

$$= \sum_{1}^{n} (4n+1)^2$$

$$= \sum_{1}^{n} 16n^2 + 8n + 1 \quad (1)$$

$$= 16\sum_{1}^{n} n^2 + 8\sum_{1}^{n} n + \sum_{1}^{n} 1$$

$$= 16\left[\frac{n}{6}(n+1)(2n+1)\right] (1) + 8\left[\frac{n}{2}(n+1)\right] (1) + n (1)$$

$$= \frac{8n}{3}(n+1)(2n+1) + 4\,n(n+1) + n \quad (1)$$

$$= \frac{8n}{3}(2n^2+3n+1) + 4n^2 + 5n \quad (1)$$

$$= \frac{1}{3}n[16n^2 + 24n + 8 + 12n + 15] \quad (1)$$

$$= \frac{1}{3}n(16n^2 + 36n + 23) \quad (1)\ [8]$$

Question 2

a) $r = \frac{\sin 2x}{\sin x}$ (1)

$$= \frac{2\sin x\cos x}{\sin x} \quad (1)$$

$$= 2\cos x \quad (1)\ [3]$$

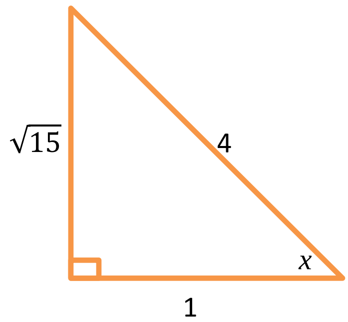
b) $S_\infty = \frac{a}{1-r}$

$$= \frac{\sin x}{1 - 2\cos x} \quad (1)$$

$$= \frac{\sin\left[\cos^{-1}\left(\frac{1}{4}\right)\right]}{1 - 2\cos\left[\cos^{-1}\left(\frac{1}{4}\right)\right]} \quad (1)$$

$$= \frac{\frac{\sqrt{15}}{4}}{1 - \frac{2}{4}} \quad (1)$$

$$= \frac{\sqrt{15}}{2} \quad (1)\ [4]$$



Question 3

a) $f(r+1) - f(r) = (r+1)! - r!$ (1)

$$= r!\,(r+1-1) \quad (1)$$

$$= r \times r! \quad (1)\ [3]$$

b) $\sum_{r=1}^{n}(r \times r!) = \sum_{r=1}^{n}(r+1)! - r!$ (1)

$$= \begin{bmatrix} 2! - 1! \\ +3! - 2! \\ +4! - 3! \\ +5! - 4! \\ \cdot \\ \cdot \\ \cdot \\ +n! - (n-1)! \\ +(n+1)! - n! \end{bmatrix} \quad (1)\ (1)$$

$$= (n+1)! - 1! \quad (1)$$

$$= (n+1)! - 1 \quad (1)\ [5]$$

Question 4

a) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$

$$y\frac{d^3y}{dx^3} (1) + \frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right) (1) + 2\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right) (1) + \frac{dy}{dx} (1) = 0$$

$$y\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right) + \frac{dy}{dx} = 0$$

$$\frac{d^3y}{dx^3} = \frac{-3\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}}{y}$$ (1) [5]

b) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$

$(1)\frac{d^2y}{dx^2} + (1)^2 + 1 = 0$ (1)

$\frac{d^2y}{dx^2} = -2$ (1)

$$\frac{d^3y}{dx^3} = \frac{-3\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right) - \frac{dy}{dx}}{y}$$

$\frac{d^3y}{dx^3} = \frac{-3(-2)(1) - 1}{1}$ (1)

$= 5$ (1)

$y = 1 + x - \frac{2}{2!}x^2 + \frac{5}{3!}x^3$ (1)

$= 1 + x - x^2 + \frac{5}{6}x^3$ (1) [6]

Question 5

a) $(1 + 3x)^{-1} =$

1(1) $+ (-1)(3x)$(1) $+ \frac{(-1)(-2)}{2!}(3x)^2$(1) $+$

$\frac{(-1)(-2)(-3)}{3!}(3x)^3$(1)

$= 1 - 3x + 9x^2 - 27x^3$(1) [4]

b) $\frac{1+x}{1+3x} = (1 + x)(1 + 3x)^{-1}$

$= (1 + x)(1 - 3x + 9x^2 - 27x^3)$ (1)

$= 1 - 3x + 9x^2 - 27x^3 + x - 3x^2 + 9x^3$ (1)

$= 1 - 2\,x + 6\,x^2 - 18\,x^3$ (1) [3]

c) $\frac{1+x}{1+3x} = \frac{101}{103}$ (1)

$103 + 103x = 101 + 303x$ (1)

$200x = 2$

$x = \frac{1}{100}$ (1)

$\frac{101}{103} = 1 - 2\left(\frac{1}{100}\right) + 6\left(\frac{1}{100}\right)^2 - 18\left(\frac{1}{100}\right)^3$ (1)

$= 0.98058$ (1) [5]

Question 6

a) 2 solutions (1) [1]

b) $f(0) = e^0 - 2\cos 0 = -1$ (1)

$f(1) = e^1 - 2\cos 1 = 1.64$

The function is **continuous** (1) and there is a **sign change** (1) so by the **IVT** (1) there is a root α in the interval [0,1]. [4]

c) $f'(x) = e^x + 2\sin x$ (1)

$x_2 = 0.5 - \frac{e^{0.5} - 2\cos 0.5}{e^{0.5} + 2\sin 0.5}$ (1) (1)

$= 0.54$ (1) [4]

d) $a = -2 \quad f(-2) = 0.967629$

$b = -1 \quad f(-1) = -0.712725$

$\beta = \frac{af(b) - bf(a)}{f(b) - f(a)}$ (1)

$= \frac{-2(-0.712725) - (-1)(0.967629)}{-0.712725 - 0.967629}$ (1)

$= -1.42$ (1) [3]