Solutions to SBA Unit 2 Test 3 (2016)

Question 1

i.

(a) no. of 4 digit numbers $= {}^{9}P_4(1) = 3024(1)$ [2]

(b) no. of odd 4 digit numbers $= 8 \times 7 \times 6 \times 5(1)$

$= 1680(1)$ [2]

ii.

(a) include at least 3 odd digits

$= ({}^{5}C_3 \times {}^{4}C_1)(1) + {}^{5}C_4(1)$

$= (10 \times 4) + 5$

$= 45(1)$

no. of choices of 4 digits $= {}^{9}C_4(1) = 126(1)$

$$\text{P(include at least 3 odd digits)} = \frac{45}{126}$$

$= \frac{5}{14}(1)$ [6]

(b) numbers that add to 28 are 9,8,7,4 or 9,8,6,5(1)

$$\text{P(add up to 28)} = \frac{2}{126}(1) = \frac{1}{63}(1) \quad [3]$$

Question 2

$$x\frac{dy}{dx} + 2y = 10x^2$$

$$\frac{dy}{dx} + \frac{2}{x}y = 10x \quad (1)$$

$$I = e^{\int \frac{2}{x}\,dx} \quad (1)$$

$$= e^{2\ln x} \quad (1)$$

$$= x^2 \quad (1)$$

$$\int \left(x^2\frac{dy}{dx} + 2xy\right)dx = \int 10x^3\,dx \ (1)$$

$$x^2y = \frac{5}{2}x^4 + c \quad (1)$$

$$y = \frac{5}{2}x^2 + \frac{c}{x^2} \quad (1) \quad [7]$$

$$3 = \frac{5}{2} + c \quad (1)$$

$$\frac{1}{2} = c \quad (1)$$

$$y = \frac{5}{2}x^2 + \frac{1}{2x^2} \quad (1) \quad [3]$$

Question 3

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-x}$$

$$m^2 + 5m + 6 = 0 \quad (1)$$

$$(m+2)(m+3) = 0 \quad (1)$$

$$m = -2 \quad m = -3 \quad (1)$$

$$C.F.\ is\ y = Ae^{-2x} + Be^{-3x} \quad (1) \quad [4]$$

$$let\ P.I.\ be\ y = pe^{-x} \quad (1)$$

$$\frac{dy}{dx} = -pe^{-x} \quad (1)$$

$$\frac{d^2y}{dx^2} = pe^{-x} \quad (1)$$

$$pe^{-x} - 5pe^{-x} + 6pe^{-x} = e^{-x} \quad (1)$$

$$2p = 1$$

$$p = \frac{1}{2} \quad (1)$$

$$P.I.\ is\ y = \frac{1}{2}e^{-x} \quad (1) \quad [6]$$

$$G.S.\ is\ y = Ae^{-2x} + Be^{-3x} + \frac{1}{2}e^{-x}(1) \quad [1]$$

$$0 = A + B + \frac{1}{2}$$

$$A + B = -\frac{1}{2} \quad (1)$$

$$\frac{dy}{dx} = -2Ae^{-2x} - 3Be^{-3x} - \frac{1}{2}e^{-x}(1)$$

$$0 = -2A - 3B - \frac{1}{2}$$

$$2A + 3B = -\frac{1}{2} \quad (1)$$

$$2A + 2B = -1$$

$$B = \frac{1}{2} \quad (1)$$

$$A + \frac{1}{2} = -\frac{1}{2}$$

$$A = -1 \quad (1)$$

$$P.S.is\ y = -e^{-2x} + \frac{1}{2}e^{-3x} + \frac{1}{2}e^{-x} \quad (1) \quad [6]$$

Question 4

(a) $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & k \end{pmatrix}$

$$|A| = -2\begin{vmatrix} 2 & 4 \\ 2 & k \end{vmatrix} + 0 - 2\begin{vmatrix} 3 & 2 \\ 4 & 2 \end{vmatrix} \quad (1)$$

$$= -2(2k - 8) - 2(6 - 8) \quad (1)$$

$$= -4k + 16 - 12 + 16 \quad (1)$$

$$= 20 - 4k \quad as\ required \quad (1) \quad [4]$$

(b) $20 - 4k = 0$ (1)

$k = 5$ (1) [2]

(c) $\begin{pmatrix} \begin{vmatrix} 0 & 2 \\ 2 & k \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 4 & k \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 4 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 4 \\ 2 & k \end{vmatrix} & \begin{vmatrix} 3 & 4 \\ 4 & k \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 4 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 4 \\ 0 & 2 \end{vmatrix} & -\begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} & \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} \end{pmatrix}$ (1) *numbers* (1) *signs*

$$\begin{pmatrix} 4 & -2k + 8 & 4 \\ -2k + 8 & 3k - 16 & 2 \\ 4 & 2 & -4 \end{pmatrix} \quad \begin{matrix} (1) \\ (1) \\ (1) \end{matrix}$$

$A^{-1} = \frac{1}{20 - 4k}$ (1) $\begin{pmatrix} 4 & 8 - 2k & 4 \\ 8 - 2k & 3k - 16 & 2 \\ 4 & 2 & -4 \end{pmatrix}$ (1) [7]

Question 5

(a) $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 3 & -1 & 3 & -2 \\ 1 & 2 & -3 & -1 \end{array}\right)$ (1) [1]

(b) $\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -4 & 6 & -2 \\ 0 & 1 & -2 & -1 \end{array}\right)$ $R_2 - 3R_1$ (1)
$R_3 - R_1$ (1)

$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -4 & 6 & -2 \\ 0 & 0 & -2 & -6 \end{array}\right)$ $4R_3 + R_2$ (1) [3]

(c) $-2z = -6$

$z = 3$ (1)

$-4y + 6z = -2$

$-4y + 18 = -2$

$y = 5$ (1)

$x + y - z = 0$

$x + 5 - 3 = 0$

$x = -2$ (1) [3]