Solutions to SBA Unit 2 Test 3 (2013)

1. i) $number\ of\ arrangements = \frac{12!}{2!2!3!4!}$ (1)

$= 831600$ (1) [2]

ii) $number\ of\ arrangements = \frac{6!}{2!4!} \times \frac{6!}{2!3!}$ (1)

$= 900$ (1) [2]

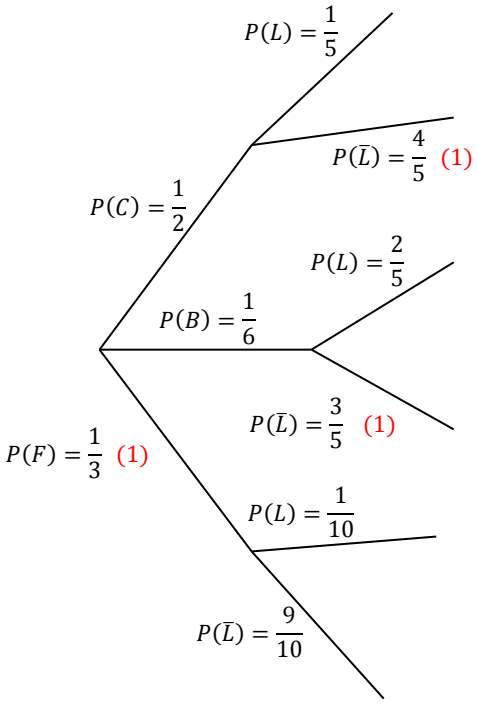
iii) $number\ of\ ways\ of\ selecting\ 4\ houses$

$= {}^{2}C_1 \times {}^{3}C_1 \times {}^{7}C_2$ (1)

$= 2 \times 3 \times 21$

$= 126$ (1) [2]

2. a)



[3]

b) (i) $P(Bill\ travels\ by\ foot\ and\ is\ late)$

$= \left(\frac{1}{3} \times \frac{1}{10}\right)$ (1)

$= \frac{1}{30}$ (1) [2]

(ii) $P(Bill\ is\ not\ late)$

$= \left(\frac{1}{2} \times \frac{4}{5}\right) + \left(\frac{1}{6} \times \frac{3}{5}\right) + \left(\frac{1}{3} \times \frac{9}{10}\right)$ (1)

$= \frac{4}{5}$ (1) [2]

c) $P(Bill\ does\ not\ travels\ on\ foot | Bill\ is\ late)$

$$= \frac{\left(\frac{1}{2} \times \frac{1}{5}\right) + \left(\frac{1}{6} \times \frac{2}{5}\right)}{1 - \frac{4}{5}} \quad (1) \quad (1)$$

$$= \frac{1}{6} \div \frac{1}{5}$$

$$= \frac{5}{6} \quad (1) \qquad [3]$$

3. a) $y = xz$

$$\frac{dy}{dx} = x\frac{dz}{dx} + z \qquad (1)$$

$$\frac{d^2y}{dx^2} = x\frac{d^2z}{dx^2} + \frac{dz}{dx} + \frac{dz}{dx} \quad (1) = x\frac{d^2z}{dx^2} + 2\left(\frac{dz}{dx}\right) \quad (1)$$

$$\frac{1}{x}\frac{d^2y}{dx^2} + \left(\frac{6}{x} - \frac{2}{x^2}\right)\frac{dy}{dx} + \left(\frac{9}{x} - \frac{6}{x^2} + \frac{2}{x^3}\right)y = 169\sin 2x$$

$$\frac{1}{x}\left(x\frac{d^2z}{dx^2} + 2\left(\frac{dz}{dx}\right)\right) + \left(\frac{6}{x} - \frac{2}{x^2}\right)\left(x\frac{dz}{dx} + z\right)$$

$$+ \left(\frac{9}{x} - \frac{6}{x^2} + \frac{2}{x^3}\right)xz = 169\sin 2x \quad (1)$$

$$\frac{d^2z}{dx^2} + \frac{2}{x}\frac{dz}{dx} + 6\frac{dz}{dx} + \frac{6z}{x} - \frac{2}{x}\frac{dz}{dx} - \frac{2z}{x^2} + 9z - \frac{6z}{x} + \frac{2z}{x^2}$$

$$= 169\sin 2x \quad (1)$$

$\frac{d^2z}{dx^2} + 6\frac{dz}{dx} + 9z = 169\sin 2x$ as required (1) [6]

b) $m^2 + 6m + 9 = 0$

$(m+3)(m+3) = 0$ (1)

$m = -3\ or\ m = -3$ (1)

$C.F.\ is\ y = (Ax + B)e^{-3x}$ (1) [3]

c) i) $z = psin\ 2x + qcos\ 2x$

$$\frac{dz}{dx} = 2pcos\ 2x - 2qsin\ 2x \quad (1)$$

$$\frac{d^2z}{dx^2} = -4psin\ 2x - 4qcos\ 2x \quad (1)$$

$$\frac{d^2z}{dx^2} + 6\frac{dz}{dx} + 9z = 169\sin 2x$$

$$-4psin\ 2x - 4qcos\ 2x + 6(2pcos\ 2x - 2qsin\ 2x) + 9(psin\ 2x + qcos\ 2x) = 169\sin 2x \ (1)$$

$$-4psin\ 2x - 4qcos\ 2x + 12pcos\ 2x - 12qsin\ 2x + 9psin\ 2x + 9qcos\ 2x = 169\sin 2x \quad (1)$$

$-4p - 12q + 9p = 169$ $\quad$ $-4q + 12p + 9q = 0$

$5p - 12q = 169$ (1) $\quad$ $12p + 5q = 0$ (1)

$$60p - 144q = 2028$$
$$60p + 25q = 0$$
$$-169q = 2028$$
$$q = -12 \quad (1)$$
$$12p - 60 = 0$$
$$p = 5 \quad (1) \qquad [8]$$

ii. $z = (Ax + B)e^{-3x} + 5\sin 2x - 12\cos 2x$ (1) [1]

iii. $-10 = (A(0) + B)e^0 + 5\sin 0 - 12\cos 0$ (1)

$$2 = B \quad (1)$$

$$\frac{dz}{dx} = -3(Ax + B)e^{-3x} + Ae^{-3x} + 10\cos 2x + 24\sin 2x \quad (1)$$

$$5 = -3(A(0) + B)e^0 + Ae^0 + 10\cos 0 + 24\sin 0 \quad (1)$$

$$-5 = -3B + A$$
$$-5 = -3(2) + A$$
$$1 = A \qquad (1)$$

$z = (x + 2)e^{-3x} + 5\sin 2x - 12\cos 2x$(1) [6]

iv. $\frac{y}{x} = (x + 2)e^{-3x} + 5\sin 2x - 12\cos 2x$ (1)

$y = x(x + 2)e^{-3x} + 5x\sin 2x - 12x\cos 2x$ (1) [2]

4. a) $\begin{vmatrix} x & 2 & 3 \\ 2 & x & 3 \\ 2 & 3 & x \end{vmatrix} = 0$

$$x\begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 3 & x \end{vmatrix} + 2\begin{vmatrix} 2 & 3 \\ x & 3 \end{vmatrix} = 0 \quad (1)$$

$$x(x^2 - 9) - 2(2x - 9) + 2(6 - 3x) = 0 \quad (1)$$

$$x^3 - 9x - 4x + 18 + 12 - 6x = 0$$

$$x^3 - 19x + 30 = 0 \quad (1)$$

$$(x - 2)(x^2 + 2x - 15) = 0 \quad (1)$$

$$(x - 2)(x - 3)(x + 5) = 0$$

$x = 2$(1) *or* $x = -5$(1) *or* $x = 3$(1) [7]

b) i) $2A + B + C = 180$ (1)

$A + 3B + 2C = 300$ (1)

$2A + B + 2C = 240$ (1) [3]

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 180 \\ 1 & 3 & 2 & 300 \\ 2 & 1 & 2 & 240 \end{array}\right) \quad (1) \qquad [1]$$

$$\begin{matrix} \\ 2r_2 - r_1 \\ r_3 - r_1 \end{matrix} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 180 \\ 0 & 5 & 3 & 420 \\ 0 & 0 & 1 & 60 \end{array}\right) \begin{matrix} \\ (1) \\ (1) \end{matrix} \qquad [2]$$

$C = 60$ (1)

$5B + 3C = 420$

$5B + 180 = 420$ (1)

$B = 48$ (1)

$2A + B + C = 180$

$2A + 48 + 60 = 180$ (1)

$A = 36$ (1) [5]