Solutions to SBA Unit 1 Test 2

1. (i) $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$

$= \frac{1-\cos 2\theta}{\sin 2\theta}$ (1)

$= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$ (1)

$= \frac{\sin\theta}{\cos\theta}$ (1)

$= \tan\theta$ as required (1) [4]

(ii) $\tan 15° = \frac{1}{\sin 2(15°)} - \frac{\cos 2(15°)}{\sin 2(15°)}$

$= \frac{1}{\sin 30°} - \frac{\cos 30°}{\sin 30°}$ (1)

$= \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ (1)

$= 2-\sqrt{3}$ as required (1) [3]

2. (i) $r = \sqrt{3^2+4^2} = 5$ (1)

$\alpha = \tan^{-1}\frac{4}{3} = 53.1°$ (1)

$3\sin\theta + 4\cos\theta = 5\sin(\theta + 53.1°)$ (1) [3]

(ii) (a) $3\sin\theta + 4\cos\theta + 1 = 0$

$5\sin(\theta+53.1°) + 1 = 0$ (1)

$\sin(\theta + 53.1°) = -\frac{1}{5}$ (1)

$\sin A = -\frac{1}{5}$ $\quad -126.9° \le A \le 233.1°$

$\sin^{-1}\left(\frac{1}{5}\right) = 11.5°$

$A = 180° + 11.5° = 191.5°$

$A = -11.5°$

$\theta = 138.4°$ (1) $\quad \theta = -64.6°$ (1) [4]

(b) $-37 \le k(3\sin\theta + 4\cos\theta) + c \le 43$

$-37 \le 5k\sin(\theta + 53.1°) + c \le 43$ (1)

$maximum\ value\ of\ \sin(\theta+53.1°) = 1$

$5k + c = 43$ (1)

$minimum\ value\ of\ \sin(\theta+53.1°) = -1$

$-5k + c = -37$ (1)

$2c = 6$

$c = 3$ (1)

$5k + 3 = 43$

$k = 8$ (1) [5]

3. (a) $x^2 + y^2 - 8x - 16y + 72 = 0$

$x^2 - 8x + y^2 - 16y + 72 = 0$

$(x-4)^2 - 16 + (y-8)^2 - 64 + 72 = 0$ (1) + (1)

$(x-4)^2 + (y-8)^2 = 8$ (1)

$centre\ (4,8)$ (1) $\quad r = \sqrt{8} = 2\sqrt{2}$ (1) [5]

(b) $distance = \sqrt{(4-0)^2 + (8-0)^2}$ (1)

$= \sqrt{80}$ (1)

$= 4\sqrt{5}$ (1) [3]

(c) $OA^2 = \left(\sqrt{80}\right)^2 - \left(\sqrt{8}\right)^2$ (1)

$= 80 - 8$

$= 72$ (1)

$OA = \sqrt{72}$

$= 6\sqrt{2}$ as required (1) [3]

4. (a) $4x^2 + 9y^2 = 36$

$When\ x = 0$

$9y^2 = 36$ (1)

$y^2 = 4$

$y = \pm 2$

$(0, -2)$ (1) $(0, 2)$ (1)

$When\ y = 0$

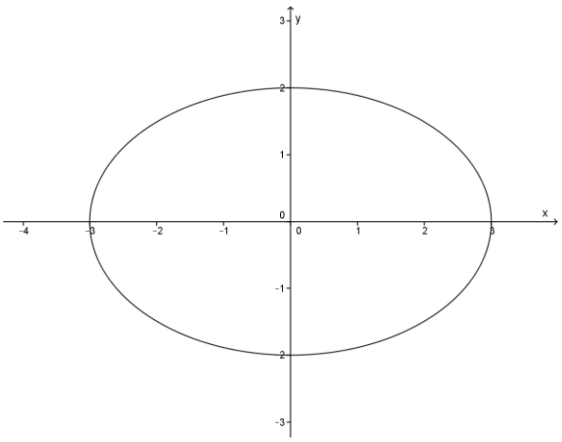
$4x^2 = 36$ (1)

$x^2 = 9$

$x = \pm 3$

$(-3, 0)$ (1) $(3, 0)$ (1) [6]

(b)Major axis = 6 (1) Minor axis = 4(1)[2]



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(c) correct intercept(1) ellipse shape (1) [2]

5. (i) $\overrightarrow{PR} = 2i + 2j + 2k$ (1)

$\overrightarrow{PQ} = -2i + 2j + 4k$ (1) [2]

(ii) $\overrightarrow{PR} \cdot \overrightarrow{PQ} = |\overrightarrow{PR}||\overrightarrow{PQ}| \cos\theta$

$2(-2) + 2(2) + 2(4)$
$= \sqrt{2^2 + 2^2 + 2^2}\sqrt{(-2)^2 + 2^2 + 4^2} \cos\theta$ (1)

$8 = \sqrt{12}\sqrt{24} \cos\theta$ (1)

$\theta = \cos^{-1} \dfrac{8}{\sqrt{12}\sqrt{24}}$ (1)

$= 61.9°$ (1) [4]

(iii) $\overrightarrow{QR} = 4i - 2k$ (1)

$Perimeter = |\overrightarrow{PR}| + |\overrightarrow{PQ}| + |\overrightarrow{QR}|$

$= \sqrt{12} + \sqrt{24} + \sqrt{4^2 + (-2)^2}$ (1)

$= 12.8\ units$ (1) [3]

6. (i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -2i - 6j + k + 3i + 4j + k$ (1)

$= i - 2j + 2k$ (1) [2]

(ii)$l_1 = 2i + 6j - k + \lambda(i - 2j + 2k)$

(1) + (1) [2]

(iv) $l_2 = \mu i + \mu k$ (1)

$l_1 = (2 + \lambda)i + (6 - 2\lambda)j + (-1 + 2\lambda)k$

$2 + \lambda = \mu$ (1)

$6 - 2\lambda = 0$ (1)

$-1 + 2\lambda = \mu$ (1)

$\lambda = 3$ (1)

$\mu = 2 + 3 = 5$ (1)

$position\ vector\ of\ C = 5i + 5k$ (1) [7]