Unit 2 Test 3

1 (a) number of possible committees (without restriction)

$= {}^{12}C_4 = 495$

number of committees with <u>no</u> females

$= {}^{8}C_4 = 70$

number of committees with <u>AT least</u> one female

$= 495 - 70 = 425$

OR

$\left({}^{8}C_3 \times {}^{4}C_1\right) + \left({}^{8}C_2 \times {}^{4}C_2\right) + \left({}^{8}C_1 \times {}^{4}C_3\right) + {}^{4}C_4 = 425$

(b) $\dfrac{9!}{2!} = 181,440$

(ii) Placing the two Es next to each other

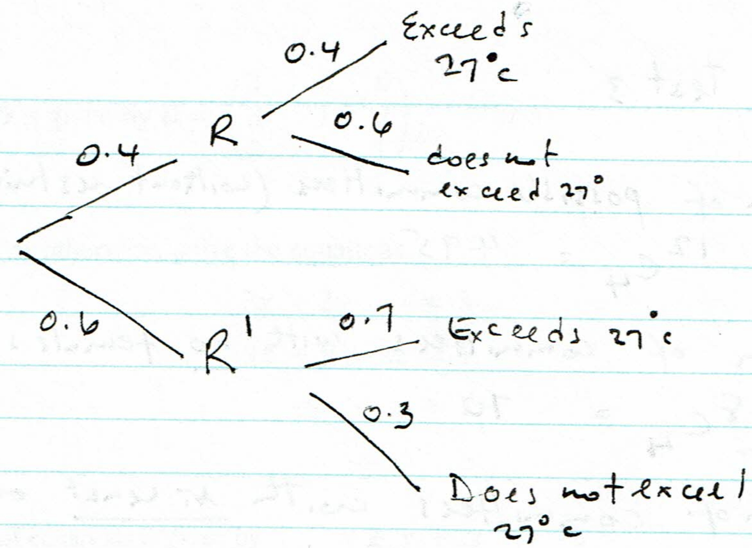
$\Rightarrow$ 8! arrangements with the E's next to each other.

Probability that E's are together

$= \dfrac{8!}{9!/2!} = 0.222 = \dfrac{2}{9}$

(c) $9 \times 17 + 9 = 162$ numbers

2



$$P(R \mid \text{Exceed } 27°c) = \frac{P(R \cap \text{exceeds } 27°c)}{P(\text{exceeds } 27°c)}$$

$$= \frac{0.4 \times 0.4}{0.4 \times 0.4 + 0.6 \times 0.7} = \frac{0.16}{0.58} =$$

$$= 0.2759$$

(b) $P(A|B) = \dfrac{P(A \cap B)}{P B}$

(i) $P(A \cap B) = P(A|B) \times P(B) = \dfrac{2}{11} \times \dfrac{11}{20} = \dfrac{1}{10}$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{2}{5} + \frac{11}{20} - \frac{1}{10} = \frac{17}{20}$$

(iii) $P(A) \times P(B) = \dfrac{2}{5} \times \dfrac{11}{20} = \dfrac{11}{50} \neq P(A \cap B)$

So A and B are NOT independent

3. $D = \begin{pmatrix} 3 & 2 & 0 \\ 3 & 1 & 2 \\ 0 & -1 & 1 \end{pmatrix}$

(i) $|D| = 3\begin{vmatrix} 1 & 2 \\ -1 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 3 & 1 \\ 0 & -1 \end{vmatrix}$

$= 3(1+2) - 2(3) = 3$

cofactor matrix $= \begin{pmatrix} 3 & -3 & +3 \\ -2 & 3 & 3 \\ 4 & -6 & -3 \end{pmatrix}$

cofactor matrix$^T = \begin{pmatrix} 3 & -2 & 4 \\ -3 & 3 & -6 \\ -3 & 3 & -3 \end{pmatrix}$

$D^{-1} = \frac{1}{3}\begin{pmatrix} 3 & -2 & 4 \\ -3 & 3 & -6 \\ -3 & 3 & -3 \end{pmatrix} = \begin{pmatrix} 1 & -2/3 & 4/3 \\ -1 & 1 & -2 \\ -1 & 1 & -1 \end{pmatrix}$

(ii) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -2/3 & 4/3 \\ -1 & 1 & -2 \\ -1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 5/3 \\ -1 \\ 0 \end{pmatrix}$

$x = \frac{5}{3}$ $y = -1$ $z = 0$

4) (i) $\begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 14 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ 6 \\ k \end{pmatrix}$

(ii) $\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 1 & 2 & 0 & k \end{array}\right)$

(iii) $\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 3 & -1 & 14 & 6 \\ 0 & 1 & -2 & k+2 \end{array}\right)$ $R_3 = R_3 - R_1$

$R_2 = R_2 - 3R_1$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & -4 & 8 & 12 \\ 0 & 1 & -2 & k+2 \end{array}\right)$

$4R_3 + R_2 = R_3$ $\left(\begin{array}{ccc|c} 1 & 1 & 2 & -2 \\ 0 & -4 & 8 & 12 \\ 0 & 0 & 0 & 4k+20 \end{array}\right)$

(iv) $4k+20 = 0$ for consistency

$k = -5$

(v) $z = \lambda$

$-4y + 8\lambda = 12 \Rightarrow y = \dfrac{12 - 8\lambda}{-4} = 2\lambda - 3$

$x + (2\lambda - 3) + 3\lambda = -2$

$x + 5\lambda - 3 = -2 \Rightarrow x = 1 - 5\lambda$

5 (a) $\frac{dy}{dx} + 5y = e^{8x}$.

$I.F. = e^{\int 5\,dx} = e^{5x}$

$ye^{5x} = \int e^{5x} e^{8x}\,dx$

$ye^{5x} = \int e^{13x}\,dx = \frac{e^{13x}}{13} + C$

$ye^{5x} = \frac{e^{13x}}{13} + C$

$y = \frac{e^{8x}}{13} + Ce^{-5x}$

when $x=0$ $y = 3/2$

$\frac{3}{2} = \frac{1}{13} + C$

$C = \frac{37}{26}$

$y = \frac{e^{8x}}{13} + \frac{37}{26} e^{-5x}$

5(b) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$

$m^2 - 6m + 9 = 0$

$(m-3)^2 = 0$

$m = 3$ (repeated)

C.F.: $y = (A + Bx)e^{3x}$

(ii) Both $y = ke^{3x}$ and kxe^{3x} are accounted for in the complementary function.

(iii) Try $y = kx^2e^{3x}$

$\frac{dy}{dx} = 3kx^2e^{3x} + 2kxe^{3x} = (3kx^2 + 2kx)e^{3x}$

$\frac{d^2y}{dx^2} = 3(3kx^2 + 2kx)e^{3x} + e^{3x}(6kx + 2k)$

$3(3kx^2 + 2kx) + (6kx + 2k) - 6(3kx^2 + 2kx) + 9kx^2 = 1$

$2k = 1 \qquad k = \frac{1}{2}$

So general solution

$y = \frac{1}{2}x^2e^{3x} + (A + Bx)e^{3x}$

$y = e^{3x}(A + Bx + \frac{1}{2}x^2)$