1(a) $S_\infty = \dfrac{a}{1-r} = 18 \Rightarrow a = 18(1-r)$ ✓

$a + ar + ar^2 = \dfrac{38}{3}$

$18(1-r) + 18(1-r)r + 18(1-r)r^2 = 38/3$ ✓

$18 - 18r^3 = \dfrac{38}{3}$ ✓

$18r^3 = 18 - \dfrac{38}{3} = \dfrac{16}{3}$

$r^3 = \dfrac{16}{18 \times 3} = \dfrac{8}{27} \Rightarrow r = 2/3$ ✓

$a = 18 \times (1 - 2/3) = 6$ ✓ [5]

(b) $a = 8 \qquad n = 23$

$5(3a + 3d) = 3a + 63d$ ✓

$12a = 48d$

$12 \times 8 = 48d$ ✓ [3]

(i) $d = 2$ ✓

(ii) $S_{15} = \dfrac{15}{2}(16 + 14(2))$ ✓ [2]

$= 330$ ✓

2 $u_1 = 2$ $\qquad u_{n+1} = 2u_n - 1$

$u_2 = 3$

$u_3 = 5$ [3]

$u_4 = 9$ $\qquad \frac{1}{2}(9-1) = 4$

(ii) $2^{n-1} + 1 = u_n$ [2]

(iii) Let $P(n)$ be the statement that
$u_n = 2^{n-1} + 1$

for $n=1$ $\quad P(1) = 2^0 + 1 = 2$ so $P(1)$ is true.

assume $P(n)$ is true for $n = k$
for $n = k+1$

$u_{k+1} = 2u_k - 1$

$= 2(2^{k-1} + 1) - 1$ [5]

$= 2^k + 2 - 1 = 2^k + 1$

So $P(k+1)$ is true when $P(k)$ is assumed true.

3 (i) $\frac{1}{r} - \frac{1}{r+2} = \frac{r+2-r}{(r+2)r} = \frac{2}{r(r+2)}$ ✓ ✓ [2]

(ii) $\frac{2}{1\times 3} + \frac{2}{2\times 4} \;\cdots\; \frac{2}{n(n+2)}$ $\quad n-1+2$

$= \left(\frac{1}{1} - \cancel{\frac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \cdots \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$

$+ \left(\frac{1}{n} - \frac{1}{n+2}\right)$

$= 1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2} = \frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}$ [5]

(iii) (a) $\sum_{r=1}^{\infty} \frac{2}{r(r+2)} = \frac{3}{2}$ [1]

(b) $\sum_{r=n+1}^{\infty} \frac{2}{r(r+2)} = \frac{3}{2} - \left(\frac{3}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right)$

$= \frac{1}{n+1} + \frac{1}{n+2}$ [2]

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

4 $f(x) = \ln(1+e^x)$

$f(0) = \ln 2$

$f'(x) = \dfrac{e^x}{1+e^x}$ $\quad f'(0) = \frac{1}{2}$

[6]

$$f''(x) = \frac{(1+e^x)e^x - e^x e^x}{(1+e^x)^2} = \frac{e^x}{(1+e^x)^2} \qquad f''(0) = \frac{1}{4}$$

$$f(x) = \ln 2 + \frac{1}{2}x + \frac{1}{4}\frac{x^2}{2!} = \ln 2 + \frac{1}{2}x + \frac{x^2}{8}$$

(ii) $$f'''(x) = \frac{(1+e^x)^2 e^x - e^x[2(1+e^x)e^x]}{(1+e^x)^4}$$

$$f'''(0) = \frac{4-4}{2^4} = 0$$

[3]

(b) $\frac{1}{2}x + \frac{1}{8}x^2$

[1]

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5 $(1+6x)^{2/3}$ up to and including the term in x^2 . [2]

$(8+6x)^{2/3}$ up to x^2 [3]

Use your answer from (b) to find an estimate for $\sqrt[3]{100}$ in the form $\frac{a}{b}$ where a and b are integers [2]

5 (a) ${}^8C_r \, x^{8-r}\left(\frac{1}{ax}\right)^r$

for the term in x, $8-2r=2$

$2r=6 \Rightarrow r=3$

So the coefficient is ${}^8C_3 \times \frac{1}{a^3} =$

$\frac{56}{a^3} = 7 \Rightarrow a^3 = 8$

$\boxed{a=2}$ [3]

5 (b) $(1+6x)^{2/3} = 1+\frac{2}{3}\times 6x + kx^2$ [2]

$= 1+4x-4x^2$

b(ii) $(8+6x)^{2/3} = 8^{2/3}\left(1+\frac{6}{8}x\right)^{2/3} = 4\left(1+\frac{6}{8}\right)^{2/3}$ [3]

$= 4+2x-\frac{1}{4}x^2$

(iii) $100 = 10^2$ $8+6x=10$ $x=\frac{1}{3}$

$\sqrt[3]{100} = 4+2\times\frac{1}{3} - \frac{1}{4}\times\left(\frac{1}{3}\right)^2 = \frac{167}{36}$ [2]

$$24x^3 + 36x^2 + 18x - 5 = 0$$

(a) $f(0.1) = -2.816$

$f(0.2) = 0.232$

[3]

So since $f(x)$ is continuous in the interval and $f(0.1) \cdot f(0.2) < 0 \Rightarrow \alpha$ lies in the interval

(b) $f(0.15) = -1.409$ (<0 so root > 0.15)

$f(0.175) \approx -0.619$ (<0 so root > 0.175)

[3]

α lies between 0.175 and 0.2

(c) $f'(x) = 72x^2 + 72x + 18$ ✓

$x_2 = 0.2 - \dfrac{f(0.2)}{f'(0.2)} =$ ✓✓

$= 0.1934$ ✓

[4]