

**HARRISON COLLEGE INTERNAL EXAMINATION  
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION  
SCHOOL BASED ASSESSMENT  
PURE MATHEMATICS  
UNIT 2 TEST 2  
1 hour 20 minutes**

This examination paper consists of **2** printed pages.

This paper consists of **8** questions.

INSTRUCTIONS TO CANDIDATES

- (i) Write **in ink**
- (ii) Write your name clearly on each sheet of paper used
- (iii) Answer **ALL** questions
- (iv) Number your questions identically as they appear on the question paper and do **NOT** **write your solutions to different questions** beside each other
- (v) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (a) Mathematical formulae
- (b) Scientific calculator (non-programmable, non-graphical)

1) A sequence of positive integers  $u_1, u_2, u_3, \dots$  is given by  $u_1 = 2$  and  $u_{n+1} = 2u_n$  for  $n \geq 1$ .

- (i) Write down the first four terms of this sequence. [1]
- (ii) State what type of sequence this is, and express  $u_n$  in terms of  $n$ . [2]

2) Dominique has been a marathon runner for many years. She ran her first marathon in approximately 5 hours. She trained intensively and each marathon she was able to decrease her time by 2.5%.

- (i) Approximately how many hours should it take Dominique to complete her 8<sup>th</sup> marathon? [2]
- (ii) How many hours in total would she have run after she completed her 8<sup>th</sup> marathon? [2]
- (iii) Her ultimate goal is to run a marathon in 4 hours. If she maintains her training schedule, how long should it take her to accomplish her goal? (Round off to the nearest marathon.) [2]

3) Use Maclaurin's Theorem to expand the function  $e^{\cos x}$ , in ascending powers of  $x$

as far as the term in  $x^2$ . [8]

4) Using the Taylor series, expand  $y$  up to terms in  $(x-1)^3$ , where  $\frac{d^2y}{dx^2} + y \frac{dy}{dx} = x$  given that  $y = 0$  and  $\frac{dy}{dx} = 1$  at  $x = 1$ . [7]

5) Without expanding  $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$  completely, find

(i) the term independent of  $x$  [4]

(ii) the coefficient of  $x^6$ . [3]

6) Given  $f(x) = e^x - 2x^2$

(i) Show that the equation  $f(x) = 0$  has a root  $\alpha$  in the interval  $[-1, 0]$ . [2]

(ii) By taking an initial approximation to  $\alpha$  to be  $-0.5$ , use the Newton-Raphson method to find a second approximation to  $\alpha$ , giving your answer correct to 3 significant figures. [4]

7) (i) Express  $\frac{1}{(r+3)(r+1)}$  in partial fractions. [4]

(ii) Hence prove, by the method of differences, that

$$\sum_{r=1}^n \frac{2}{(r+3)(r+1)} = \frac{n(5n+13)}{6(n+2)(n+3)} \quad [5]$$

8) Prove by the method of mathematical induction that  $(1+x)^n \geq 1 + nx$  for  $n \geq 0$ . [7]

### ANSWERS

1) (i) 2, 4, 8, 16 (ii) geometric;  $u_n = 2^n$

2) (i) 4.2 hrs (ii) 36.7 hrs (iii) approx 10 marathons

3)  $e - \frac{e^2}{2} x^2$

4)  $f(x) = (x-1) + \frac{(x-1)^2}{2!}$

5) (i)  $\frac{7}{18}$  (ii)  $\frac{189}{16}$

6)  $-0.541$

7)  $\frac{1}{2(r+1)} - \frac{1}{2(r+3)}$

8)  $(1+x)^k + x(1+x)^k$  vs.  $(1+x) + xk$