HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2015
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 TEST 1
1 hour 20 minutes

This test paper consists of **2** printed pages.
This paper consists of **9** questions.
The maximum mark for this test is **53**.

INSTRUCTIONS TO CANDIDATES

(i) Write **in ink**
(ii) Write your name clearly on each sheet of paper used
(iii) Answer **ALL** questions
(iv) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other
(v) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

1) Given that $\boldsymbol{p}$ and $\boldsymbol{q}$ are propositions, use the algebra of propositions to simplify fully
$(\boldsymbol{p} \wedge \boldsymbol{q}) \vee (\boldsymbol{p} \wedge \sim \boldsymbol{q})$ [3]
Total 3 marks

2) Express $\dfrac{5\sqrt{2}+1}{2-\sqrt{2}}$ in the form $p + q\sqrt{2}$ where p and q are real numbers. [5]
Total 5 marks

3) Evaluate $\sum_{r=5}^{200}(3r-2)$. [4]
Total 4 marks

4) Prove by mathematical induction that $3^{2n} - 1$ is divisible by 8 for all $n \in \boldsymbol{Z}^+$. [6]
Total 6 marks

5) (i) The cubic polynomial $2x^3 + 5x^2 + px - 6$, where p is a constant is denoted by $f(x)$.
Given that $(x + 2)$ is a factor of $f(x)$, find the value of p. [2]
(ii) When p has this value, find all the roots of the equation $f(x) = 0$. [6]
Total 8 marks

PTO

6) The population of a village at the beginning of the year 2005 was 410.
The population increased so that, after a period of n years, the new population was $410(1.06)^n$. Calculate estimates of

(i) the population at the beginning of 2015 [2]

(ii) the year in which the population is expected to first reached 900. [5]

Total 7 marks

7) The function f is given by f: $x \rightarrow e^{2x}$, $x \in \boldsymbol{R}$ and the function g is given by g: $x \rightarrow \ln 2x$, $x \in \boldsymbol{R}$, $x > 0$.

(i) Find an expression in terms of x for $f^{-1}(x)$. [4]

(ii) State for $f^{-1}(x)$

(a) the domain [1]

(b) the range. [1]

(iii) Determine $fg(x)$, simplifying your answer. [3]

Total 9 marks

8) Find the range of values of $x \in \boldsymbol{R}$ for which $\dfrac{3x-2}{x-3} \leq 2$, $x \neq 3$ [5]

Total 5 marks

9) Solve for x, $x^2 - 4|x| + 4 = 0$. [6]

Total 6 marks

End of Test

HARRISON COLLEGE – BARBADOS
SOLUTIONS – CAPE 2015: UNIT 1 TEST 1

1) $(\boldsymbol{p} \wedge \boldsymbol{q}) \vee (\boldsymbol{p} \wedge \sim \boldsymbol{q})$

$= \boldsymbol{p} \wedge (\boldsymbol{q} \vee \sim \boldsymbol{q})$	distributive	[1 mark]
$= \boldsymbol{p} \wedge 1$	complement	[1 mark]
$= \boldsymbol{p}$	identity	[1 mark]

2) $\dfrac{5\sqrt{2}+1}{2-\sqrt{2}}$

$= \dfrac{(5\sqrt{2}+1)}{(2-\sqrt{2})} \times \dfrac{(2+\sqrt{2})}{(2+\sqrt{2})}$ [1 mark]

$= \dfrac{(10\sqrt{2}+10+2+\sqrt{2})}{(4-2)}$ [1 mark]

$= \dfrac{12\sqrt{2}+12\sqrt{2})}{2}$ [1 mark]

$= 6 + 6\sqrt{2}$ [1 + 1 marks]

3) $\sum_{r=5}^{200}(3r-2)$.

$= \sum_{r=1}^{200}(3r-2) - \sum_{r=1}^{4}(3r-2)$ [1 mark]

$= 3[\sum_{r=1}^{200} r - \sum_{1}^{200} 2] - 3[\sum_{r=1}^{4} r - \sum_{1}^{4} 2]$ [1 mark]

$= 3\left[\frac{1}{2}(200)(200+1) - (2 \times 200)\right] - 3\left[\frac{1}{2}(4)(4+1) - (2 \times 4)\right]$ [1 mark]

$= [59\ 900] - [22]$

$= 59\ 878$ [1 mark]

4) Let P_n be the statement "$3^{2n} - 1$ is divisible by 8 $\forall\, n \in Z^+$."

Basic Step – To Prove P_n is true for $n = 1$ i.e. To Prove P_1 is true

When $n = 1$; $P_1 = 3^2 - 1 = 8$ which is divisible by 8.
$\therefore$ P_1 is true. [1 mark]

Inductive Step – Assume P_n is true for $n = k$ i.e. Assume P_k is true
i.e. $P_k \equiv 3^{2k} - 1$ is divisible by 8 $\forall\, k \in Z^+$. [1 mark]

We are required to show that $P_{k+1} \equiv 3^{2(k+1)} - 1 = 3^{2k+2} - 1$ is divisible by 8.

Now $P_{k+1} - P_k = [3^{2k+2} - 1] - [3^{2k} - 1]$ [1 mark]

$= 3^{2k+2} - 3^{2k}$

$= 3^{2k}[3^2 - 1]$

$= 3^{2k}[8]$ [1 mark]

$\therefore P_{k+1} = P_k + 3^{2k}[8]$

P_k is divisible by 8 by assumption and $3^{2k}[8]$ is also divisible by 8

$\therefore P_{k+1}$ is divisible by 8. [1 mark]

Conclusion

$P_1 \Rightarrow P_2$

$P_2 \Rightarrow P_3$

$P_{n\text{-}1} \Rightarrow P_n$

Hence, by MI, P_n is true $\forall\, n \in Z^+$ [1 mark]

5) (a) (i) Let $f(x) = 2x^3 + 5x^2 + px - 6$

$f(-2) = 0$

$2(-2)^3 + 5(-2)^2 + p(-2) - 6 = 0$ [1 mark]

$-2 = 2p$

$-1 = p$ [1 mark]

$f(x) = 2x^3 + 5x^2 - x - 6$

$2x^2 + x - 3$ [1 mark] Quotient

$(x + 2)\overline{)\,2x^3 + 5x^2 - x - 6}$

Subtract $\underline{2x^3 + 4x^2}$

$x^2 - x$

Subtract $\underline{x^2 + 2x}$ [1 mark]

$-3x - 6$

Subtract $\underline{-3x - 6}$

0 0

So $f(x) = 0 \Rightarrow (x + 2)(2x^2 + x - 3) = 0$

$(x + 2)(2x + 3)(x - 1) = 0$ [1 mark]

$x = -2, -\dfrac{3}{2}, 1$ [1 + 1 + 1 marks]

6) (i) Year 2010 i.e. $n = 0$, $P = 410$

Let $P(n) = 410\,(1.06)^n$

$P(10) = 410\,(1.06)^{10}$ [1 mark]

$= 734.2$ persons [1 mark]

accept 734

(ii) $P(n) = 410(1.06)^n$

$900 = 410(1.06)^n$

$\frac{900}{410} = (1.06)^n$ [1 mark]

$\ln\left(\frac{900}{410}\right) = \ln[(1.06)^n]$ [1 mark]

$\ln\left(\frac{900}{410}\right) = n\ln(1.06)$ [1 mark]

$\frac{\ln\left(\frac{900}{410}\right)}{\ln(1.06)} = n$

13.5 years = n [1 mark]

Beginning of year 2005 + 13.5 years = 2018 [1 mark]

7) (i) $f: x \rightarrow e^{2x}, x \in R$; $g: x \rightarrow \ln 2x, x \in \boldsymbol{R}, x > 0$

Let $y = f(x)$

$y = e^{2x}$

$x = e^{2y}$ [1 mark]

$\ln x = \ln(e^{2y})$ [1 mark]

$\ln x = 2y$ [1 mark]

$\frac{1}{2}\ln x = y$

$\frac{1}{2}\ln x = f^{-1}(x)$ [1 mark]

(ii) (a) Domain of f^{-1}, $x \in \boldsymbol{R}, x > 0$ [1 mark]

(b) Range of f^{-1}, $f(x) \in \boldsymbol{R}$ [1 mark]

(iii) $fg(x) = f(\ln 2x)$ [1 mark]

$= e^{2\ln 2x}$

$= e^{\ln(2x)^2}$ [1 mark]

$= 4x^2$ [1 mark]

8) $\frac{3x-2}{x-3} \leq 2$, $x \neq 3$

$\frac{3x-2}{x-3} \times (x-3)^2 \leq 2 \times (x-3)^2$ [1 mark]

$(3x-2)(x-3) \leq 2x^2 - 12x + 18$ [1 mark]

$x^2 + x - 12 \leq 0$ [1 mark]

$(x+4)(x-3) \leq 0$

$-4 \leq \boldsymbol{x} < 3$ [1 + 1 marks]

9) Solve for x, $x^2 - 4|x| + 4 = 0$.

$x^2 - 4(x) + 4 = 0$ [1 mark]
$(x - 2)^2 = 0$ [1 mark]
$x = 2$ twice [1 mark]

OR

$x^2 - 4(-x) + 4 = 0$ [1 mark]
$x^2 + 4x + 4 = 0$
$(x + 2)^2 = 0$ [1 mark]
$x = -2$ twice [1 mark]