HARRISON COLLEGE INTERNAL EXAMINATION, March 2020
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 2 - TEST 1 Preview

TIME: 1 Hour & 20 minutes

This examination paper consists of 2 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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1. (a) The complex numbers z and w are given by $z = 1 + i$ and $w = 2 - 5i$ respectively. Find.

(i) $2z + w$ [1]

(ii) $|2z + w|$ [2]

(iii) $\arg(2z + w)$ [2]

(iv) $\frac{w}{z}$ giving your answer in the form $x + iy$ [4]

(b) (i) Express $\sin n\theta$ and $\cos n\theta$ in terms of $e^{in\theta}$ and $e^{-in\theta}$. [2]

(ii) Hence show that

$$cos^3\theta = \frac{1}{8}(2cos3\theta + 6cos\theta)$$ [5]

TOTAL 16 marks

2. (a) Find $\frac{dy}{dx}$ when

(i) $y = e^{2x} + sin^{-1}(2x)$ [3]

(ii) $y = \frac{\ln(\sqrt{x})}{cos^{-1}x}$ [3]

(b) Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point (1, 2). [4]

(c) A curve is defined by the parametric equations

$$y = t - 3 \text{ and } x = t^2 - 6t + 4$$

Find the gradient of the curve at the point for which $t = 2$. [5]

(d) Let $f(x, y) = (x^2 + y^2)^2 + e^{xy}$, find $\frac{\partial^2 f}{\partial x \partial y}$ [2]

TOTAL 17 marks

3. (a) (i) Express $f(x) = \frac{2x+1}{(x-3)^2}$ in partial fractions. [5]

(ii) Hence find the exact value of $\int_4^{10} f(x)dx$. [5]

(b) Using the substitution $u = x^4$, find

$$\int_0^2 \frac{x^3}{1+x^8}\, dx$$ (give your answer to 2 decimal places) [5]

(c) It is given that for $n \geq 0$, $I_n = \int_0^1 e^{-x}x^n\, dx$

(i) Show that for $n \geq 1$

$$I_n = nI_{(n-1)} - e^{-1}$$ [4]

(ii) Find the exact value of I_3 . [4]

(d) Use the trapezium rule with 4 trapezia of equal width to estimate the value of

$\int_2^3 \sqrt{1 + x^2}\, dx$. Give your answer to 2 decimal places. [4]

TOTAL 27 marks

END OF EXAMINATION

Answers

1 (a) (i) $4 - 3i$

(ii) 5

(iii) -0.644 rads

(iv) $-\frac{3}{2} - \frac{7}{2}i$

2 (a) (i) $2e^{2x} + \frac{2}{\sqrt{1-4x^2}}$

(ii) $\frac{\frac{1}{2x}cos^{-1}x + \frac{ln\sqrt{x}}{\sqrt{1-x^2}}}{(cos^{-1}x)^2}$

(b) -2

(c) $\frac{-1}{2}$

(d) $8xy + e^{xy}(xy + 1)$

3. (a) (i) $\frac{2}{x-3} + \frac{7}{(x-3)^2}$

(ii) $2ln7 + 6$

(b) 0.38

(c) (ii) $6 - 16e^{-1}$

(d) 2.70