

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2017

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 3

1 HOUR 20 MINUTES

This examination paper consists of 3 printed pages.

This paper consists of 7 questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

- (i) Write your name clearly on each sheet of paper used
- (ii) Answer **ALL** questions
- (iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other.
- (iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (a) Mathematical formulae
- (b) Scientific calculator (non-programmable, non-graphical)

1. Given that $A = \begin{pmatrix} 3 & 0 & 2 \\ -1 & 2 & -1 \\ 2 & 4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 8 & -4 \\ -1 & -1 & 1 \\ -8 & -12 & 6 \end{pmatrix}$,

(a) Show that A is a non-singular matrix. [3]

(b) Determine AB [3]

(c) Hence, deduce A^{-1} , the inverse of A . [2]

(d) Use A^{-1} to solve the system of equations

$$3x + 2z = 11$$

$$-x + 2y - z = 0$$

$$2x + 4y + z = 15$$

[3]

2. Given the following system of equations

$$x + y - z = 6$$

$$2x - y + 3z = -2$$

$$2y + z = 5$$

(a) Write the system as an augmented matrix. [1]

(b) Use row reduction to solve the system of equations. [5]

(c) Determine the general solution if the last equation of the system was replaced by $2x + 2y - 2z = 12$. [3]

3. (a) (i) Solve the differential equation $\frac{dy}{dt} = y \sin t$ to obtain y in terms of t . [4]

(ii) Given that $y = 50$ when $t = \pi$, show that $y = 50e^{-(1+\cos t)}$. [2]

(b) A wave machine at a leisure pool produces waves. The height of the water, y cm, above a fixed point at time t seconds is given by the differential equation $\frac{dy}{dt} = y \sin t$.

Given that this height is 50 cm after π seconds, find, to the nearest centimeter, the height of the water after 6 seconds. [2]

4. Find the general solution of the differential equation

$$\frac{dy}{dx} + 2y \cot x = \sin x, \quad 0 < x < \frac{\pi}{2}$$

giving your answer in the form $y = f(x)$. [7]

5. Given that for the differential equation

$$\frac{d^2x}{dt^2} + 5 \frac{dx}{dt} + 6x = 2e^{-t}$$

$x = 0$ and $\frac{dx}{dt} = 2$ at $t = 0$.

Find x in terms of t . [8]

6. A committee of 6 people, which must contain at least 4 men and at least 1 woman, is to be chosen from 10 men and 9 women.

(i) Find the number of possible committees that can be chosen. [3]

(ii) Find the probability that one particular man, Simon, and one particular woman, Diana, are both on the committee. [2]

(iii) Find the number of possible committees that include either Simon or Diana but not both. [3]

(iv) The committee that is chosen consists of 4 men and 2 women. They queue up randomly in a line for refreshments. Find the probability that the women are not next to each other in the queue. [3]

7. (i) The digits of the number 1 244 687 can be rearranged to give many different 7 - digit numbers. How many of these 7 - digit numbers are even. [4]

(ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, 2, 4, 6, 7, 8? [2]

End of Test