1. $(p \wedge q) \vee (p \wedge \sim q)$

$= p \wedge (q \vee \sim q)$

$= p \wedge T$

$= p$ [3]

2. (i) $\sum_{1}^{n+1} r + 2 = \sum_{r=1}^{n+1} r + \sum_{1}^{n+1} 2$

$= \frac{n+1)(n+2)}{2} + 2(n+1)$

$= \frac{1}{2}(n+1)(n+6)$ [3]

(ii) $\frac{1}{2}(n+1)(n+6) = 7n$

$(n+1)(n+6) = 14n$

$n^2 + 7n + 6 = 14n$

$n^2 - 7n + 6 = 0$ [3]

$(n-1)(n-6) = 0 \implies n = 1 \quad n = 6$

(b) $\frac{3\sqrt{2}+5}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}}$

$= \frac{6\sqrt{2}+10-6-5\sqrt{2}}{4-2} = \frac{\sqrt{2}+4}{2}$

$= 2+\frac{1}{2}\sqrt{2}$ [3]

$x=2 \quad y=1/2$

3 (i) let $f(x) = x^3 + px^2 + x + q$

$f(-1) = -1 + p - 1 + q = 0$ ✓

$p+q = 2$ ✓

$f(2) = 8 + 4p + 2 + q = 0$ ✓

$4p + q = -10$ ✓

Solve simultaneously

$p+q = 2$
$4p+q = -10$

$3p = -12 \implies p = -4$ ✓

$-4 + q = 2 \implies q = 6$ ✓ [6]

3 (ii) $f(3) = 27 - 36 + 3 + 6 = 0$

So $x = 3$ is another root [3]

(4) Let $P(n)$ be the statement that

$$\sum_{1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

for $n = 1$

$$\frac{1}{(2-1)(2+1)} = \frac{1}{3} = \frac{1}{2+1} = \frac{1}{3}$$ ✓

so $P(1)$ is true

Assume $P(n)$ is true for $n = k$ ✓

now for $n = k+1$

$$\sum_{1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \sum_{1}^{k} \frac{1}{(2r-1)(2r+1)} + \frac{1}{(2k+1)(2k+3)}$$ ✓

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$ ✓

$$= \frac{k(2k+3) + 1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2 + 3k + 1)}{(2k+1)(2k+3)} = \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$ ✓ [7]

$$\frac{k+1}{2k+3} = \frac{k+1}{2(k+1)+3}$$ ✓

So $P(k+1)$ is true when $P(k)$ is true etc. ✓

5. $6e^{2y} = 7e^{y} + 3$

$6e^{2y} - 7e^{y} - 3 = 0$

Let $u = e^{y}$

$6u^2 - 7u - 3 = 0$

$(3u+1)(2u-3) = 0$

$u = -1/3 \quad u = 3/2$

$e^{y} = -1/3 \qquad e^{y} = 3/2$

no solution

$y = \ln 3/2$

[5]

6. $P(n) = 2400\ (1.06)^{n}$

(i) $n = 10$; $P(10) = 2400\ (1.06)^{10}$

$= 4298.03$

≈ 4298

[3]

(ii) $2400\ (1.06)^{n} = 7000$

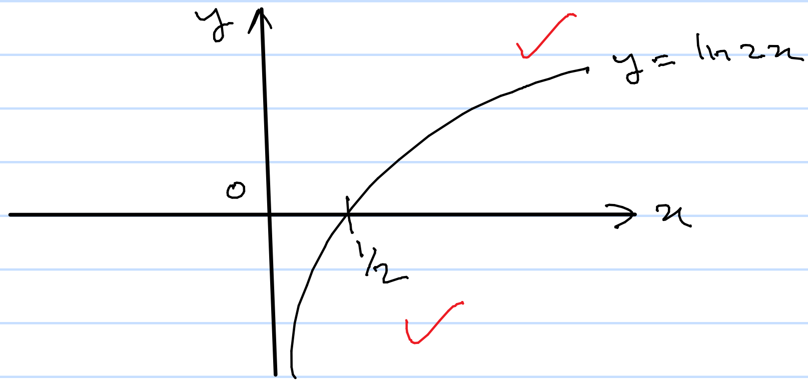
$1.06^{n} = \dfrac{7000}{2400}$

$n = \dfrac{\ln\left(\frac{7000}{2400}\right)}{\ln 1.06} = 18.37$

[4]

so population will reach 7000 in the year 2018.

7. (i)



[2]

(ii) $f(x) = \ln 2x$

$x = \ln 2y$

$2y = e^x$

$y = \frac{e^x}{2} \Rightarrow f^{-1}(x) = \frac{e^x}{2}$

[4]

(iii) (a) $\mathbb{R}$ (The set of Real numbers) [1]

(b) $y > 0$ [1]

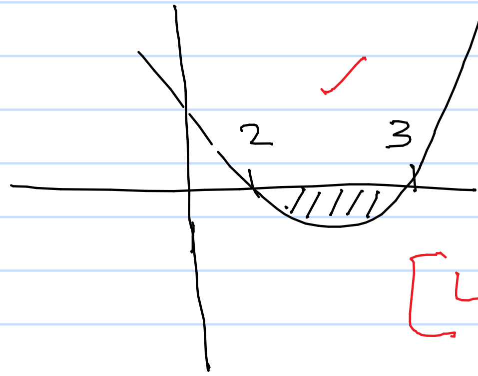
iv $gf(x) = e^{2\ln 2x} = e^{\ln(2x)^2} = e^{\ln 4x^2}$

$= 4x^2$ [3]

8 $\quad \dfrac{x-2}{x-3} \le 0$

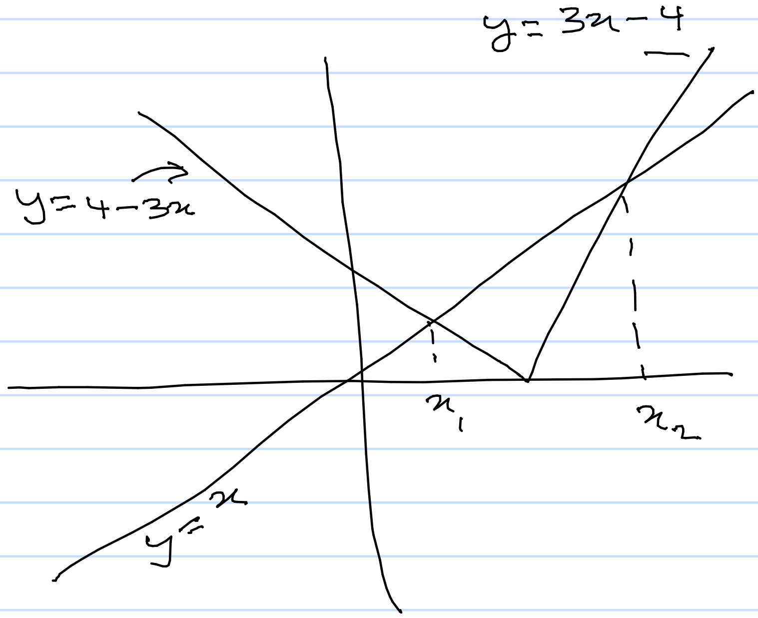
$(x-3)(x-2) \le 0$

$2 \le x \le 3$



[4]

9. $\quad |4-3x| < x$



<u>to find x_1</u>

$4-3x = x$

$4x = 4$

$x_1 = 1$

<u>to find x_2</u>

$3x - 4 = x$

$2x = 4$

$x = 2$

So solution is $\quad 1 \le x \le 2$

[5]