HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 1
1 hour 20 minutes

This examination paper consists of 2 pages.
This paper consists of 6 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. The complex number $7 + 3i$ is denoted by z.

   Find

   i. $|z|$ [1]

   ii. $\arg z$ [1]

   iii. $\frac{z}{4-i}$ [5]

   Total 7 marks

2. The loci C_1 and C_2 are given by $\arg(z - 2 - 2i) = \frac{1}{4}\pi$ and $|z| = |z - 10|$ respectively

   i. Sketch on a single Argand diagram the loci C_1 and C_2. [3]

   ii. Indicate, by shading, the region of the Argand diagram for which

   $0 \leq \arg(z - 2 - 2i) \leq \frac{1}{4}\pi$ and $|z| \geq |z - 10|$ [2]

   Total 5 marks

3. Given that

$$x = \sec^2 3y \ , \qquad 0 < y < \frac{\pi}{6}$$

   a) Find $\frac{dx}{dy}$ in terms of y. [4]

b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

[4]

Total 8 marks

4. Given that

$$f(x) = \frac{25}{(3+2x)^2(1-x)} \ , |x| < 1$$

a) Express $f(x)$as a sum of partial fractions [6]

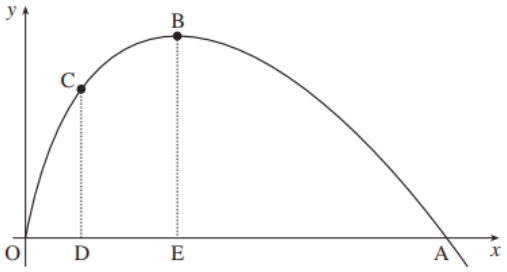
b) Hence find $\int f(x)\ dx$ [3]

Total 9 marks

5. The curve

$$y = 2x - x\ln x, where\ x > 0$$

shown below, crosses the *x*-axis at A, and has a turning point at B. The point C on the curve has *x*-coordinate 1. Lines CD and BE are drawn parallel to the *y*-axis.



Not to scale

i. Find the *x*-coordinate of A, giving your answer in terms of e. [3]

ii. Find the exact coordinates of B. [8]

iii. Show that the tangents at A and C are perpendicular to each other. [4]

iv. Using integration by parts, show that

$$\int x\ln x\ dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + c$$

Hence find the exact area of the region enclosed by the curve, the *x*-axis and the lines CD and BE. [8]

Total 23 marks

6. Use the substitution $u = 2^x$ to find the exact value of

$$\int_0^1 \frac{2^x}{(2^x+1)^2}\ dx$$

[8]

Total 8 marks

END OF EXAMINATION