HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT PREVIEW
PURE MATHEMATICS
UNIT 2 – TEST 2
1 hour 20 minutes

This examination paper consists of 3 pages.
This paper consists of 6 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. (a) The sum of an infinite geometric sequence is $13\frac{1}{2}$ and the sum of the first three terms is 13. Find the first term. [a = 9] [4]

   (b) An arithmetic series, with first term 12 and common difference *d*, consists of 23 terms. Given that the sum of the last 3 terms is 5 times the sum of the first 3 terms. Find:

   (i) the value of *d*. [d = 3] [3]

   (ii) the sum of the first 17 terms. [612] [2]

   Total 9 marks

2. The sequence $u_1, u_2, u_3 \ldots$ is defined by $u_1 = 3$ and $u_{n+1} = 3u_n - 2$.

   (i) Find u_2 and u_3 and verify that $\frac{1}{2}(u_4 - 1) = 27$ [7, 19] [3]

   (ii) Hence suggest an expression in terms of *n* for u_n. [$2(3^{n-1}) + 1$] [2]

   (iii) Use induction to prove that your answer to part (ii) is correct. [5]

   Total 10 marks

3. (i) Show that $\frac{1}{2r-3} - \frac{1}{2r+1} = \frac{4}{4r^2-4r-3}$. [2]

(ii) Hence, find an expression, in terms of n, for

$$\sum_{r=2}^{n} \frac{4}{4r^2-4r-3}$$

$\frac{4}{3} - \frac{4n}{(2n-1)(2n+1)}$ [6]

(ii) Determine

$$\sum_{r=2}^{\infty} \frac{4}{4r^2-4r-3}$$

$\left[\frac{4}{3}\right]$ [1]

Total 9 marks

4. The function *f* is defined by $f(x) = ln\left(\frac{1}{1-x}\right)$

(a) Write down the value of the constant term in the Maclaurin series for $f(x)$. [0] [1]

(b) Find the first three derivatives of $f(x)$ and hence show that the Maclaurin series for $f(x)$ up to and including the x^3 term is $x + \frac{x^2}{2} + \frac{x^3}{3}$. [6]

(c) Use this series to find an approximate value for $\ln 2$. [$\frac{2}{3}$] [3]

Total 10 marks

5. (a) The coefficient of x in the expansion of $\left(x + \frac{1}{ax^2}\right)^7$ is $\frac{7}{3}$.

Find the possible values of a. [± 3] [3]

(b) (i) Expand $(2+x)^{-2}$ in ascending powers of x up to and including the term in x^3.

$\frac{1}{4} - \frac{x}{4} + \frac{3x^2}{16} - \frac{1}{8}x^3$ [4]

(ii) State the values of x for which the expansion is valid. [$-2 < x < 2$] [1]

(iii) Hence find the coefficient of x^3 in the expansion of $\frac{(1+x^2)}{(2+x)^2}$. $\left[-\frac{3}{8}\right]$ [2]

Total 10 marks

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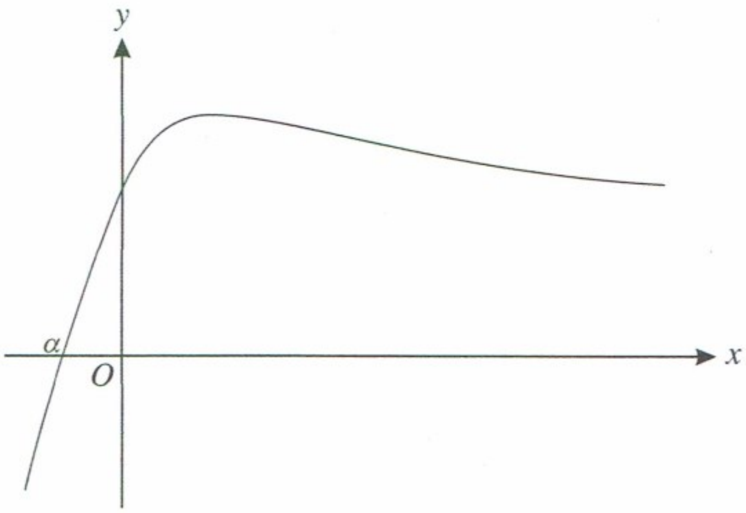
6.



The diagram shows the curve with equation $y = xe^{-x} + 1$. The curve crosses the x-axis at $x = \alpha$.

(i) Show that the equation $xe^{-x} + 1 = 0$ has a root α in the interval $[-1, 0]$. [3]

(ii) Use differentiation to show that the x-coordinate of the stationary point is 1. [2]

α is to be found using the Newton-Raphson method, with $f(x) = xe^{-x} + 1$.

(iii) Explain why this method will not converge to α if an initial approximation x_1 is chosen such that $x_1 > 1$. [2]

(iv) Use this method, with a first approximation $x_1 = 0$, to find the next three approximations x_2, x_3 and x_4. Find α, correct to 3 decimal places. [5]

[-0.567] **Total 12 marks**

END OF EXAMINATION