HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2017
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 2
1 hour 20 minutes

This examination paper consists of 2 pages.
This paper consists of 6 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1) a) Write down the n^{th} term of the sequence $5, 9, 13, 17, \ldots$ [1]

b) Hence, show that the sum of the first *n* terms of the series $5^2 + 9^2 + 13^2 + 17^2 + \cdots$ is given by

$\frac{1}{3}n(16n^2 + 36n + 23)$ [8]

Total 9 marks

2) The first three terms of a geometric sequence are $\sin x$, $\sin 2x$ and $4 \sin x \cos^2 x$,

a) Find the common ratio r, in its simplest form. [3]

Given that $x = \cos^{-1}\left(\frac{1}{4}\right)$ $, x > 0$

b) Show that the sum to infinity of the series is $\frac{\sqrt{15}}{2}$. [4]

Total 7 marks

3) a) Given that

$$f(r) = r!$$

show that

$$f(r+1) - f(r) = r \times r!$$

[3]

b) Hence find

$$\sum_{r=1}^{n} (r \times r!)$$

[5]

Total 8 marks

PLEASE TURN OVER

4) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y = 0$

a) Find an expression for $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y. [5]

Given that $y = 1$ and $\frac{dy}{dx} = 1$ at $x = 0$.

b) Find the power series for *y*, in ascending powers of *x*, up to and including the term in x^3. [6]

Total 11 marks

5) $f(x) = (1 + 3x)^{-1}$, $|x| < \frac{1}{3}$.

a) Expand $f(x)$ in ascending powers of *x* up to and including the term in x^3. [5]

b) Hence show that,

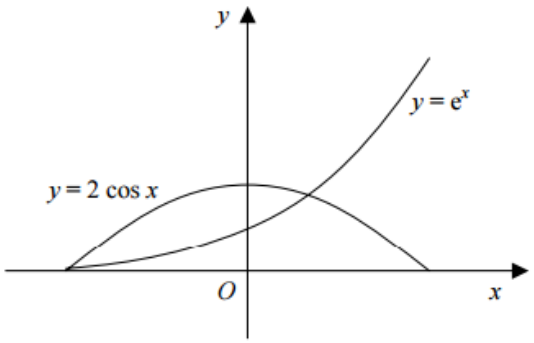
$$\frac{1+x}{1+3x} \approx 1 - 2x + 6x^2 - 18x^3$$

[3]

c) By taking a suitable value for *x*, which should be stated, use the series expansion in part b) to find an approximate value for $\frac{101}{103}$, giving your answer to 5 decimal places. [5]

Total 13 marks

6) The figure below shows the graph of $y = 2\cos x$ and $y = e^x$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



Given that $f(x) = e^x - 2\cos x$

a) Write down the number of solutions of the equation $f(x) = 0$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [1]

b) Show that the equation $f(x) = 0$ has a solution, α, in the interval $[0, 1]$. [4]

c) Using 0.5 as a first approximation to α, use the Newton-Raphson process once to find an improved estimate for α, giving your answer correst to 2 decimal places. [4]

There is another root, β, of the equation $f(x) = 0$ in the interval $[-2, -1]$.

d) Use linear interpolation once on this interval to estimate the value of β, giving your answer correct to 2 decimal places. [3]

Total 12 marks

END OF EXAMINATION