HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2017

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS

UNIT 2 – TEST 1

1 HOUR 20 MINUTES

This examination paper consists of 3 printed pages.

This paper consists of **7** questions.

The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES

(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other.
(iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

(a) Mathematical formulae

(b) Scientific calculator (non-programmable, non-graphical)

1. Differentiate with respect to x.

   (a) $x\sin^{-1}\left(\frac{x}{2}\right)$ [3]

   (b) $\frac{\ln(x^2+1)}{x}$ [3]

2. A curve C has equation $2^x + y^2 = 2xy$

   Find the exact value of $\frac{dy}{dx}$ at the point on C with coordinates $(3, 2)$. [7]

3. A curve C has parametric equations

$$x = \sin^2 t\,, \quad y = 2\tan t\,, \qquad 0 \le t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t. [4]

The tangent to C at the point where $t = \frac{\pi}{3}$ cuts the x-axis at the point P.

(b) Find the x-coordinate of P. [6]

4. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \; dx = e(e - 1)$$

[6]

5.

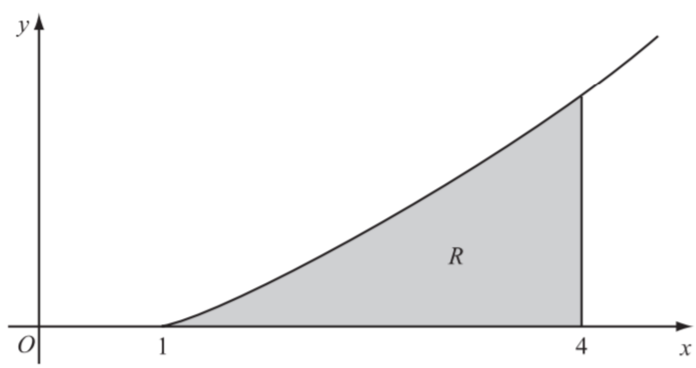


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x\ln x\,, x \ge 1$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 4$.

The table below shows corresponding values of x and y for $y = x\ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

(a) Complete the table, giving your answers to 3 decimal places. [2]

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 2 decimal places. [4]

(c) (i) Use integration by parts to find $\int x\ln x \; dx$.

(ii) Hence find the exact area of R, giving your answer in the form $\frac{1}{4}(a\ln 2 + b)$ where a and b are integers. [7]

6. Given that $z = \sqrt{3} - i$

(a) Show that $\frac{z}{z^*} = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ [3]

(b) Find the value of $\left|\frac{z}{z^*}\right|$ [1]

(c) Verify, for $z = \sqrt{3} - i$, that $\arg\left(\frac{z}{z^*}\right) = \arg z - \arg z^*$ [4]

(d) Display on a single Argand diagram z, z^* and $\frac{z}{z^*}$. [3]

7. Given that $3 + i$ is a root of the equation $f(x) = 0$, where

$$f(x) = 2x^3 + ax^2 + bx - 10 \quad a, b \in \mathbb{R}$$

(a) find the other two roots of the equation $f(x) = 0$, [4]

(b) find the value of a and the value of b. [3]

END OF EXAMINATION

QUESTION 1 SOLUTION

(a)

$$\sin^{-1}\left(\frac{x}{2}\right) + \frac{x}{2\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

(b)

$$\frac{\left[\left(\frac{2x}{x^2+1}\right)x - \ln(x^2+1)\right]}{x^2}$$

QUESTION 2 SOLUTION

$$4\ln 2 - 2 = \frac{dy}{dx}$$

QUESTION 3 SOLUTION

(a) $x\,\frac{dy}{dx} = \frac{2\sec^2 t}{2\sin t\cos t}$

(b)

$$x = \frac{3}{8}$$

QUESTION 4 SOLUTION

$$-e^1 + e^2$$

QUESTION 5 SOLUTION

(a) 1.386, 2.291

(b) $A \quad = \frac{1}{4}[29.47]$

≈ 7.37

(c) (i)

$$\frac{x^2}{2}\ln x - \frac{x^2}{4} + c$$

(d)

$$\frac{1}{4}(64\ln 2 - 15)$$

QUESTION 6 SOLUTION

(a)

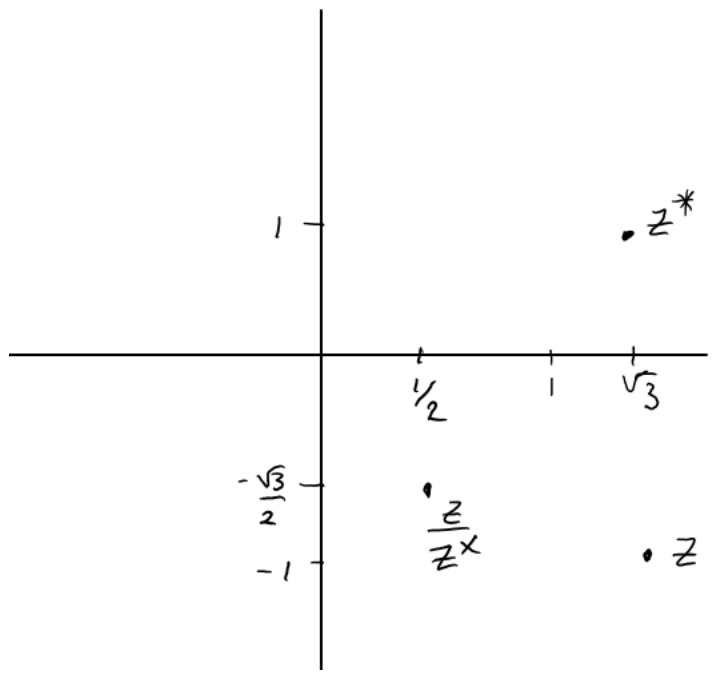
$$\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

(b) 1

(c)

$$-\frac{\pi}{3}$$

(d)



QUESTION 7 SOLUTION

(a) $\gamma = \frac{1}{2}$

(b) Type equation here.

$a = -13$

(c) $26 = b$