HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2016
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 2
1 hour 20 minutes

This examination paper consists of 2 pages.
This paper consists of 5 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. A geometric progression has first term $\log_2 27$ and common ratio $\log_2 y$.
   (a) Find the set of values of y for which the geometric progression has a sum to infinity. [2]
   (b) Find the **EXACT** value of y for which the sum to infinity of the geometric progression is3. [7]

Total 9 marks

2. (a) Write down the first four terms of Maclaurin expansion for $\ln(1+x)$. [1]

   (b) Hence, determine the first four terms for the expansion of $\ln(1+x^2)$. [2]

   (c) By using your result from (b) find the **EXACT** value of

$$1-\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{3}\left(\frac{1}{2}\right)^4-\frac{1}{4}\left(\frac{1}{2}\right)^6+\cdots$$

[5]

Total 8 marks

3. (a) Express $\frac{1}{r(r+2)}$ in partial fractions. [5]

(b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{1}{r(r+2)} = \frac{n(an+b)}{4(n+1)(n+2)}$$

where *a* and *b* are constants to be found. [8]

(c) Hence show that

$$\sum_{r=n+1}^{2n} \frac{1}{r(r+2)} = \frac{n(4n+5)}{4(n+1)(n+2)(2n+1)}$$

[5]

Total 18 marks

4. (a) Find the binomial expansion of $\sqrt{1-8x}$, in ascending powers of *x* up to and including the term in x^3, simplifying each term. [4]

(b) State the values of *x* for which the expansion is valid. [1]

(c) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and obtain an approximation to $\sqrt{23}$. Give your answer correct to **5** decimal places. [5]

Total 10 marks

5. (a) Show that $\frac{d}{dx}(2^x) = 2^x \ln 2$ [4]

(b) Given that $f(x) = 2^x + x - 4$

i. Show that the equation $f(x) = 0$ has a root α in the interval[1,2]. [4]

ii. Use linear interpolation in the interval [1,2] to find an approximation to α. Give your answer as an **EXACT** value. [3]

iii. Taking $x_1 = 1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to **3** decimal places. [4]

Total 15 marks

END OF EXAMINATION