HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2015
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 1
1 hour 20 minutes

This examination paper consists of 3 pages.
This paper consists of 5 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write in ink.
2. Write your name clearly on each sheet of paper used.
3. Answer **ALL** questions.
4. Do **NOT** do questions beside one another.
5. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

---

1. A botanist is studying the regeneration of an area of moorland following a fire. The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M.

   Model A is given by

$$M = 900 - \frac{1500}{3t+2}$$

   Model B is given by

$$M = 900 - \frac{1500}{2 + 5\ln(t+1)}$$

   (a) For each model, find to 3 significant figures

      i. the value of M when t = 3 [2]

      ii. the rate at which the biomass is increasing when t = 3. [8]

Total 10 marks

2. Given the experimental heat equation $u(x,t) = e^{-k^2 t}\sin x$, where k is a constant

   (a) Find

      i. $\frac{\partial u}{\partial t}$ [1]

      ii. $\frac{\partial^2 u}{\partial x^2}$ [2]

(b) Hence determine if the experimental equation satisfies the theoretical heat equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$ [2]

Total 5 marks

3. (a) Differentiate $xtan^{-1}x$ with respect to x. [3]

(b) Hence, show that $\int_0^1 \tan^{-1} x = \frac{\pi}{4} - \ln \sqrt{2}$ [6]

Total 9 marks

4. (a) Copy and complete the table below for the equation $y = \frac{2 \sin 2x}{1+\cos x}$. Give your answers to 5 decimal places. [2]

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157		0

(b) Use the trapezium rule, with all the values of y in the completed table, to solve

$$\int_0^{\frac{\pi}{2}} \frac{2 \sin 2x}{1 + \cos x}\, dx$$

giving your answer to 4 decimal places. [3]

(c) Using the substation $u = 1 + \cos x$ show that

$$\int \frac{2 \sin 2x}{1 + \cos x}\, dx = 4 \ln(1 + \cos x) - 4 \cos x - 4 + c$$ [6]

(d) Hence calculate the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{2 \sin 2x}{1 + \cos x}\, dx$$ [2]

(e) State, to 2 significant figures, the difference between the exact value in (d) and the approximate value in (b). [1]

Total 14 marks

PLEASE TURN OVER

5. (a) Use DeMoivre Theorem to prove that $\sin 4\theta = 4\cos^3\theta \sin\theta - 4\cos\theta \sin^3\theta$

[4]

(b) Given the complex numbers $z_1 = 2 + 3i$, $z_2 = 3 + 2i$, $z_3 = \boldsymbol{a} + \boldsymbol{b}i$ where $a, b \in \mathbb{R}$

(i) Find the exact value of $|z_1 + z_2|$ in the form $x\sqrt{2}$. [4]

Given that $\boldsymbol{w} = \frac{z_1 z_3}{z_2}$

(ii) find $\boldsymbol{w}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$, giving your answer in the form $x + iy$, $x, y \in \mathbb{R}$ [6]

Given also that $\boldsymbol{w} = \frac{17}{13} - \frac{7}{13}i$

(iii) find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$. [6]

(iv) find $\arg \boldsymbol{w}$, giving your answer in radians to 3 decimal places. [2]

Total 22 marks

END OF EXAMINATION