HARRISON COLLEGE INTERNAL EXAMINATION 2014
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 2
1 hour 30 minutes

This examination paper consists of 2 pages.
This paper consists of 5 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. A sequence is given by

$$x_1 = 1$$
$$x_{n+1} = x_n(p + x_n)$$

where p is a constant ($p \neq 0$).

(i) Find x_2 in terms of p. [1]

(ii) Show that $x_3 = 1 + 3p + 2p^2$ [2]

Given that $x_3 = 1$,

(iii) find the value of p. [3]

(iv) write down the value of x_{28}. [1]

Total: 7 marks

2. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(i) the common ratio [4]

(ii) the first term [2]

(iii) the sum to infinity [2]

(iv) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. [6]

Total: 14 marks

3.(i) Obtain the first three terms of the binomial expansion of $(1 + 4x^2)^{\frac{1}{2}}$ in ascending powers of x. [3]

(ii) State the range of values of x for which the full expansion is valid. [2]

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(iii) By integrating the three terms in your expansion, find an approximate value for

$$\int_0^{\frac{1}{4}} (1+4x^2)^{\frac{1}{2}}\, dx$$ [3]

Total: 8 marks

4. (a) Given that

$$f(r) = (r-1)r(r+1)(r+2)$$

show that

$$f(r+1) - f(r) = kr(r+1)(r+2)$$

stating the value of the constant k. [3]

(b) Use the method of differences to find

$$\sum_{r=1}^{n} r(r+1)(r+2)$$

giving your answer in factorized form. [4]

(c) Prove by mathematical induction that, for all positive integers n,

$$\sum_{r=1}^{n} \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$ [10]

(d) Hence find

$$\sum_{r=1}^{\infty} \frac{2}{r(r+1)(r+2)}$$ [2]

Total: 19 marks

5. The temperature θ °C of a room t hours after a heating system has been turned on is given by

$$\theta = t + 26 - 20e^{-0.5t}, \quad t \geq 0.$$

The heating system switches off when $\theta = 20$. The time $t = \alpha$, when the heating system switches off, is the solution of the equation $\theta - 20 = 0$, where α lies in the interval $[1.8, 2]$.

(a) Using the end points of the interval $[1.8, 2]$, find, by linear interpolation once, an approximation to α. Give your answer to 2 decimal places. [5]

(b) Taking 1.9 as a first approximation to α, use the Newton-Raphson procedure once to obtain a second approximation to α. Give your answer to 3 decimal places. [6]

(c) Use your answer to part (*b*) to find, to the nearest minute, the time for which the heating system was on. [1]

Total: 12 marks

END OF EXAMINATION