HARRISON COLLEGE INTERNAL EXAMINATION 2014
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 1
1 hour 30 minutes

This examination paper consists of 2 pages.
This paper consists of 3 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. (a) Given that $y^3 + 3y = x^3$, show that

$$\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$$ [5]

(b) Given that $f(x, y) = e^{xy}$, find f_{xy}. [3]

(c) The curve C is defined parametrically by

$$x = 4\sin^2 t \ , \ \ y = \frac{2}{\sec t} \qquad 0 \le t \le \frac{\pi}{2}$$

The point $P(3\ , 1)$ lies on the curve ***C.***

i. Obtain an expression for $\frac{dy}{dx}$ in terms of t. [4]

ii. Find the value of t at the point P. [2]

iii. Find the equation of the normal to the curve **C** at P. [3]

Total: 17 marks

2. (a) Let $f(x) = \frac{4x}{(3x+1)(x+1)^2}$

i. Express $f(x)$ in partial fractions. [5]

ii. Hence show that $\int_0^1 f(x)\ dx = 1 - \ln 2$. [5]

PLEASE TURN OVER

(b) i. Show that, using the substitution $x = e^u$

$$\int \frac{2 + \ln x}{x^2}\, dx = \int (2 + u)e^{-u}\, du$$ [3]

ii. Hence, or otherwise, evaluate in terms of e

$$\int_1^e \frac{2 + \ln x}{x^2}\, dx$$ [7]

(c) Use the trapezium rule with 4 strips to find an approximation to

$$\int_1^3 \frac{1}{x^3 + 3}\, dx$$

giving your answer to 3 significant figures. [3]

Total: 23 marks

3. (a) The complex number $1 + i\sqrt{3}$ is denoted by u.

i. Express u in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $-\pi < \theta \le \pi$. [3]

ii. Hence, or otherwise, find the modulus and argument of u^2. [2]

iii. Show that u is a root of the equation $z^2 - 2z + 4 = 0$. [2]

iv. Hence state the other root. [1]

(b) The point *P* represents a complex number *z* on an Argand diagram, where

$$|z + 1 + 2i| = \sqrt{2}|z - 1|$$

i. Show that the locus of *P* is a circle, stating the coordinates of the centre and the radius of this circle. [7]

The point *Q* represents a complex number *z* on an Argand diagram, where

$$\tan[arg\,(z + 1)] = 1$$

ii. On the same Argand diagram, sketch the locus of *P* and the locus of *Q*. [4]

iii. On your diagram, shade the region which satisfies both

$$|z + 1 + 2i| > \sqrt{2}|z - 1| \text{ and } \tan\,[arg\,(z + 1)] > 1.$$ [1]

Total: 20 marks

END OF EXAMINATION