

**HARRISON COLLEGE INTERNAL EXAMINATION 2013**  
**CARIBBEAN ADVANCED PROFICIENCY EXAMINATION**  
**SCHOOL BASED ASSESSMENT**  
**PURE MATHEMATICS**  
**UNIT 2 – TEST 2**  
**1 hour 30 minutes**

This examination paper consists of 2 pages.

This paper consists of 4 questions.

The maximum marks for this examination is 60.

**INSTRUCTIONS TO CANDIDATES**

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

**EXAMINATION MATERIALS ALLOWED**

1. Mathematical formulae sheet
  2. Scientific Non-programmable calculator (non-graphical)
- 

1. a) Express  $\frac{1}{r(r+2)}$  in partial fractions. [5]

b) Hence prove, by the method of differences, that

$$\sum_{r=1}^n \frac{4}{r(r+2)} = \frac{n(3n+5)}{(n+1)(n+2)} \quad [10]$$

c) Find, to 4 decimal places, the value of

$$\sum_{r=50}^{100} \frac{4}{r(r+2)} \quad [3]$$

**Total: 18 marks**

2. Given that  $f(x) = \tan x$ ,

a) Find

i.  $f\left(\frac{\pi}{4}\right)$  [1]

ii.  $f'\left(\frac{\pi}{4}\right)$  [2]

iii.  $f''\left(\frac{\pi}{4}\right)$  [3]

**PLEASE TURN OVER**

iv.  $f''' \left( \frac{\pi}{4} \right)$  [4]

b) Find the Taylor series expansion of  $\tan x$ , in ascending powers of  $\left(x - \frac{\pi}{4}\right)$ , up to and including the term in  $\left(x - \frac{\pi}{4}\right)^3$ . [2]

c) Hence, show that

$$\tan \left( \frac{3\pi}{10} \right) \approx 1 + \frac{\pi}{10} + \frac{\pi^2}{200} + \frac{\pi^3}{3000} \quad [3]$$

**Total: 15 marks**

3. a) Find the first four terms, in ascending powers of  $x$ , in the binomial expansion of  $(2 + kx)^7$ , where  $k$  is a non-zero constant. Give each term in its simplest form. [2]

b) Given that, in this expansion, the coefficients of  $x^2$  is 6 times the coefficient of  $x$ , find

i. the value of  $k$  [3]

ii. the coefficient of  $x$  [1]

iii. the coefficient of  $x^2$  [1]

iv. the coefficient of  $x^3$  [1]

v. Rewrite the expansion in part **a** replacing the  $k$  and simplify. [1]

c) Hence evaluate  $(2.02)^7$  correct to 2 decimal places. [4]

**Total: 13 marks**

4. The equation  $24x^3 + 36x^2 + 18x - 5 = 0$  has one real root,  $\alpha$ .

a) Show that  $\alpha$  lies in the interval  $0.1 < x < 0.2$ . [5]

b) Starting in the interval  $0.1 < x < 0.2$ , use interval bisection twice to obtain an interval of width 0.025 within which  $\alpha$  must lie. [5]

c) Taking  $x_1 = 0.2$  as a first approximation to  $\alpha$ , use the Newton-Raphson method to find a second approximation,  $x_2$ , to  $\alpha$ . Give your answer to four decimal places. [4]

**Total: 14 marks**