HARRISON COLLEGE INTERNAL EXAMINATIONS APRIL 2013 : PURE MATHEMATICS [UNIT2 – TEST 1]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & comments
1.(a)(i)	$(3+i)$	1 Total = 1 mark
(a) (ii)	$(3+i)+(3-i)=-p$ $p=-6$ $(3+i)(3-i)=q$ $q=10$	1 1 1 1 Total = 4 marks
(b)(i)	$\sqrt{3}+i=r(cos\theta+isin\theta)$ $r=\sqrt{(\sqrt{3})^2+1^2}=2$ $tan\theta=\frac{1}{\sqrt{3}}\Rightarrow\theta=\frac{\pi}{6}$ $\sqrt{3}+i=2\left(cos\frac{\pi}{6}+isin\frac{\pi}{6}\right)$ $(\sqrt{3}+i)^5=2^5(cos\frac{5\pi}{6}+isin\frac{5\pi}{6})$ $=32\left(-\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right)=-16\sqrt{3}+16i$	 1 1 1 1 1 Total = 5 marks
(c) (i)	$\|z-3-4i\|^2=9$ $\|z-3-4i\|=3$ $\Rightarrow$ locus: circle centre (3, 4) radius 3 units	1 1 Total = 2 marks
(c)(ii)	$\|z-1\|=\|z-i\|$ $\Rightarrow$ locus: perpendicular bisector of the line joining (0,1) and (1,0) OR Locus: the line *x* = *y*	1 1 Total =2 marks

c(iii)	$(x+iy)+(x-iy)=4$ $x=2$	 1 1 Total =2 marks
(d)	$e^{i\theta} = cos\theta + isin\theta$ $cos\theta = \dfrac{e^{i\theta}+e^{-i\theta}}{2}$ $2cos\theta = e^{i\theta}+e^{-i\theta}$ $(2cos\theta)^5 = (e^{i\theta}+e^{-i\theta})^5$ $32\ COS^5\theta\ = e^{i5\theta} + 5e^{i3\theta} + 10e^{i\theta} + 10e^{-i\theta} + 5e^{-i3\theta} + e^{-5\theta}$ $= 2cos5\theta + 10\cos 3\theta + 20cos\theta$ $= \frac{1}{16}\ (cos5\theta + 5\cos 3\theta + 10cos\theta)$	1 1 1 1 1 Total = 5 marks
2 (a)(i)	$y = e^{x^3} + tan^{-1}x$ $\dfrac{dy}{dx} = e^{x^3} \times \ldots$ $= e^{x^3} \times 3x^2 + \cdots$ $\dfrac{dy}{dx} = e^{x^3} \times 3x^2 + \dfrac{1}{1+x^2}$	 1 1 1 Total = 3 marks
2 (a) (ii)	$y = \dfrac{lnx}{sin^{-1}x}$ $\dfrac{dy}{dx} = \dfrac{\ldots + \cdots}{(sin^{-1}x)^2}$ $= \dfrac{sin^{-1}x \times \frac{1}{x} + \ \ldots}{(sin^{-1}x)^2}$ $= \dfrac{sin^{-1}x \times \frac{1}{x} + \ lnx \times \frac{1}{\sqrt{1-x^2}}}{(sin^{-1}x)^2}$	 1 [attempting use of quotient or product rule) 1 [for $\frac{d(lnx)}{dx}$] 1 [for $\frac{d(sin^{-1}x)}{dx}$] Total = 3 marks

2(b)	$4y - x = xy$ $4\frac{dy}{dx} - 1 = x\frac{dy}{dx} + y$ $\frac{dy}{dx} = \frac{1+y}{4-x}$ $\frac{dy}{dx} = 4$ when $x = 3$ $y = 3$ Equation of tangent: $y - 3 = 4(x - 3)$	1 1 1-[using student's expression for $\frac{dy}{dx}$] 1 Total = 4marks
2(c) (i)	$y = t^3 - 3t \quad \Rightarrow \frac{dy}{dt} = 3t^2 - 3$ $x = 2t \quad \Rightarrow \frac{dx}{dt} = 2$ $\frac{dy}{dx} = \frac{3t^2 - 3}{2}$	1 1 1 Total = 3 marks
2(c)(ii)	$\frac{d^2y}{dx^2} = \frac{\frac{d(\frac{dy}{dt})}{dt}}{\frac{dx}{dt}}$ $\Rightarrow \frac{3t}{2}$	1 [for dividing student's $\frac{d^2y}{dt^2}$ by $\frac{dx}{dt}$] 1 [c.a.o] Total = 2 marks
2(d)	$f(x, y) = 4x - 5x^2y^3 + 2y^2$ $\frac{\partial f}{\partial x} = 4 - 10xy^3$ $\frac{\partial^2 f}{\partial x \partial y} = -30xy^2$	1 1 Total = 2 marks

3(a)(i)	$\frac{x^3 - 3x^2 + 4}{x - 2} = x^2 - x - 2$ $x^2 - x - 2 = (x - 2)(x + 1)$ $x^3 - 3x^2 + 4 = (x - 2)^2(x + 1)$	1 1 Total = 2 marks
3(a)(ii))	$\frac{3}{x^3 - 3x^2 + 4} = \frac{A}{(x + 1)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$ $A = \frac{1}{3}$ $B = -\frac{1}{3}$ $C = 1$	1 1 1 1 Total = 4 marks

3(a)(iii)	$\int f(x)dx = \int \frac{1}{3(x+1)} - \frac{1}{3(x-2)} + \frac{1}{(x-2)^2}\ dx$ $= \quad \frac{ln\lvert x+1\rvert}{3} \quad + \cdots$ $= \quad \frac{ln\lvert x+1\rvert}{3} - \frac{ln\lvert x-2\rvert}{3} + \cdots$ $= \frac{ln\lvert x+1\rvert}{3} - \frac{ln\lvert x-2\rvert}{3} - (x-2)^{-1} + \text{constant}$	 1 1 1 Total = 3 marks
3b(i)	$I_n = \int_1^e (lnx)^n\ dx$ Use integration by parts with $u = (lnx)^n$ and $\frac{dv}{dx} = 1$ $I_n = (lnx)^n \int_1^e 1\ dx - \int_1^e xn(lnx)^{n-1}\frac{1}{x}\ dx$ $= e - n\int_1^e (lnx)^{n-1}\ dx$ $= e - nI_{n-1}$	 1 1 1 1 Total = 4 marks
3 (b) (ii)	$I_3 = 3 - 3\,I_2$ $= e - 3(e - 2I_1)$ $= 4e - 6I_0$ $I_0 = \int_1^e dx = e - 1$ $I_3 = 4e - 6(e-1) = 6 - 2e$	 1 1 1 1 Total= 4marks
3(c)	$\int_0^3 \sqrt{x^2+3}\,dx$ $= \frac{1}{2}\cdot 1 \cdot [\sqrt{3} + \sqrt{12} + 2(\sqrt{4} + \sqrt{7})]$ = 7.24	1 [for correct interval =1] 1 [for using 4 ordinates] 1 [for correct 'y' values] 1 [for correct use of trapezium rule] 1 [C.A.O] Total = 5 marks