

HARRISON COLLEGE INTERNAL EXAMINATION, APRIL 2013
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 2 - TEST 1

TIME: 1 Hour & 30 minutes

This examination paper consists of 2 printed pages.
The paper consists of 3 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
 2. Electronic calculator (non-programmable, non-graphical)
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1. (a) One root of the quadratic equation $z^2 + pz + q = 0$, where p and q are real, is the complex number $(3 - i)$.
 - (i) Write down the other root. [1]
 - (ii) Find the values of p and q . [4]
- (b) Use De Moivre's theorem to express $(\sqrt{3} + i)^5$ in the form $(-a\sqrt{3} + ai)$ where a is a real number. [5]
- (c) Sketch and describe the following loci in separate Argand diagrams:
 - (i) $|z - 3 - 4i|^2 = 9$ [2]
 - (ii) $|z - 1| = |z - i|$ [2]
 - (iii) $z + z^* = 4$ where z^* is the conjugate of z . [2]
- (d) By expressing $\cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that
$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5 \cos 3\theta + 10\cos\theta)$$
 [5]

TOTAL 21 marks

2. (a) Find $\frac{dy}{dx}$ when:
- (i) $y = e^{x^3} + \tan^{-1}(x)$ [3]
- (ii) $y = \frac{\ln x}{\sin^{-1}x}$ [3]

- (b) The equation of a curve is given by

$$4y - x = xy$$

Find the equation of the tangent to the given curve at the point (3, 3). [4]

- (c) A curve is defined by the parametric equations

$$y = t^3 - 3t \text{ and } x = 2t$$

Find in terms of t

- (i) $\frac{dy}{dx}$ [3]
- (ii) $\frac{d^2y}{dx^2}$ [2]
- (d) Let $f(x, y) = 4x - 5x^2y^3 + 2y^2$, find $\frac{\partial^2 f}{\partial x \partial y}$ [2]

TOTAL 17 marks

3. (a) (i) Given that $(x - 2)$ is a factor of $x^3 - 3x^2 + 4$ completely factorise

$$x^3 - 3x^2 + 4$$
 [2]

- (ii) Express $f(x) = \frac{3}{x^3 - 3x^2 + 4}$ in partial fractions. [4]

- (iii) Hence find $\int f(x) dx$ [3]

- (b) It is given that for non-negative integers n ,

$$I_n = \int_1^e (\ln x)^n dx$$

- (i) Show that for $n \geq 1$

$$I_n = e - nI_{(n-1)}$$
 [4]

- (ii) Find I_3 in terms of e . [4]

- (c) Use the trapezium rule with 3 intervals of equal width to estimate the value of

$$\int_0^3 \sqrt{x^2 + 3} dx \text{ . Give your answer to 2 decimal places.} [5]$$

TOTAL 22 marks