PURE MATHEMATICS
PREVIEW UNIT 2 - TEST 1

1. (a) One root of the quadratic equation $z^2 + pz + q = 0$, where p and q are real, is the complex number $(2 + 3i)$.

(i) Write down the other root. [1]

(ii) Find the values of p and q. [4]

(b) Use de Moivre's theorem to obtain the roots of the equation $z^3 = 8$ in Cartesian form. [5]

(c) If $z = x + iy$ and $z^* = x - iy$ where $x, y \in R$, find

(i) the equation of the circle in the x-y plane which is given by

$$\left|z - 2 - i\sqrt{3}\right| = \sqrt{2}\left|z^* - 1 + i\sqrt{3}\right| \quad [4]$$

(ii) the centre and radius of this circle. [2]

(d) By expressing $cos\theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$, show that

$$cos^4\theta = \frac{1}{8}(cos4\theta + 4\cos 2\theta + 3) \quad [5]$$

2. (a) Find $\frac{dy}{dx}$ when:

(i) $y = e^{cosx} + sin^{-1}(x)$ [3]

(ii) $y = \frac{tan^{-1}x}{lnx}$ [3]

(b) The equation of a curve is given by

$$4x - y^2 = xy$$

Find the equation of the tangent to the given curve at the point (5, -2). [4]

(c) A curve is defined by the parametric equations

$$y = t^3 \text{ and } x = t^2 + t$$

Find in terms of t

(i) $\frac{dy}{dx}$ [3]

(ii) $\frac{d^2y}{dx^2}$ [4]

(d) Let $f(x, y) = 4x^2 - 5x^3y + 3y^3$ find $\frac{\partial^2 f}{\partial x \partial y}$ [2]

3. (a) (i) Express $f(x) = \frac{x-1}{(x-2)^2(x+1)}$ in partial fractions. [4]

(ii) Hence find $\int f(x)dx$ [3]

(b) It is given that for non-negative integers n,

$$I_n = \int_0^{\frac{\pi}{2}} x^n cosx\ dx$$

(i) Show that for $n \geq 2$

$$I_n = (\frac{\pi}{2})^n - n(n-1)I_{(n-2)}$$ [5]

(ii) Find I_4 in terms of π. [4]

(c) Use the trapezium rule with 3 intervals of equal width to estimate the value of

$$\int_0^3 \log(x^2+9)\ dx$$ [4]

Answers:

1. (a) (i) $2-3i$ (ii) $p=-4\ \ q=13$

(b) 2, $-1+i\sqrt{3}$, $-1-i\sqrt{3}$

(c) (i) $x^2+(y-\sqrt{3})^2=2$ (ii) centre $(0,\sqrt{3})$ radius = $\sqrt{2}$

2. (a) (i) $-sinx\ e^{\cos x} + \frac{1}{\sqrt{1-x^2}}$ (ii) $\frac{lnx(\frac{1}{1+x^2)} - tan^{-1}x\ (\frac{1}{x})}{(\ln x)^2}$

(b) $(y+2)=6(x-5)$

(c) (i) $\frac{3t^2}{2t+1}$ (ii) $\frac{6t^2+6t}{(2t+1)^3}$

(d) $-15x^2$

3. (a) (i) $\frac{-2}{9(x+1)} + \frac{2}{9(x-2)} + \frac{1}{3\ (x-2)^2}$ (ii) $\frac{-2}{9}\ln(x+1) + \frac{2}{9}\ln(x-2) - \frac{1}{3}(x-2)^{-1} + c$

(b) (ii) $\frac{\pi^4}{16} - 3\pi^2 + 24$

(c) 3.22