HARRISON COLLEGE INTERNAL EXAMINATIONS 2012 : PURE MATHEMATICS [UNIT2 – TEST: 3]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & comments
1.(a)	${}^{12}C_6$ $= 924$	1 1 Total = 2
(b)	${}^{5}C_2 \times {}^{4}C_2 \times {}^{3}C_2$ $= 180$ So required probability = $\frac{180}{924}$	1 1 1 Total = 3
(b)	${}^{5}C_3 \times {}^{7}C_3$ $= 350$ So required probability = $\frac{350}{924}$	1 1 1 Total =3

2 (i)	$\frac{8!}{2! \times 2!}$ $= 10\ 080$	1 [for 8!] 1 [for division by $2! \times 2!$] Total =2
(ii)	$\frac{7!}{2!}$ $= 2520$	1 [for 7!] 1 [for division by 2!} Total =2
(iii)	Pr (B and B'): $\left(\frac{2}{8} \times \frac{6}{7}\right)$ + Pr(B' and B): $\left(\frac{6}{8} \times \frac{2}{7}\right)$ + Pr(B and B): $\left(\frac{2}{8} \times \frac{1}{7}\right)$ Sum of above $= \frac{26}{56} = \frac{13}{28}$	1 1 1 1 (correct answer only) Total =4

3(a)	Since A and B are independent $\Longrightarrow P(A \cap B) = P(A) \times P(B)$ Let P(B) = *x* $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $0.7 = 0.6 + x - 0.6x$ $x = 0.25$	 1 1 1 1 Total =4
(b)	Probability that A or B occurs but not both $= 0.6 + 0.25 - 2 \times (0.6 \times 0.25)$ $= 0.55$	 1 1 Total = 2

4 (a)	$\frac{(2-3i)^2}{2+i} = \frac{(2-3i)\times(2-3i)}{2+i} \times \frac{2-i}{2-1}$ $= \frac{-5-12i}{2+i} \times \frac{2-i}{2-i}$ $= \frac{-22-19i}{5}$ $= -\frac{22}{5} - \frac{19}{5}i$	1 1 1 1 Total = 4
(b) (i)	If $3-5i$ is a root $\Rightarrow$ $3+5i$ is a root as well. The sum of the roots = 6 The product of the roots :$(3-5i)(3+5i) = 34$ So equation is $z^2 - 6z + 34 = 0$	1 1 1 1 Total = 4
(ii)	$\frac{z^3 - z^2 + 4z + 170}{z^2 - 6z + 34} = z + 5$ So $z = -5$; $z = 3 - 5i$; $z = 3 + 5$ are solutions.	1 1 1 Total = 3

5 (i)	$1+\ i\sqrt{3}\ \Rightarrow\ =R(cos\alpha+i\ sin\alpha)$; $R=\ \sqrt{(1)^2+\ (\sqrt{3})^2}=2$ $\alpha=\ tan^{-1}(\frac{\sqrt{3}}{1})=\frac{\pi}{3}$ So $1+\ i\sqrt{3}\ \ =2(cos\frac{\pi}{3}+i\ sin\frac{\pi}{3})$	1 1 1 Total = 3
(ii)	$(1+\ i\sqrt{3})^5=(2(cos\frac{\pi}{3}+i\ sin\frac{\pi}{3}))^5$ = 32($\cos\frac{5\pi}{3}+i\ sin\frac{5\pi}{3}$) = $16-27.7\ i$	1 1 Total = 2

6 (i)	Locus of points satisfying $\|z+6\| = \|z-4i\|$ - the perpendicular bisector of the line joining - the points $(-6,0)$ and $(0,4)$	 1 1 1 Total = 3
(ii)	The locus of the points satisfying $\|z-1+4i\| = 3$ is : circle centre $(1,-4)$ radius = 3 units	 1 1 1 Total =3

7 (i)	$\begin{pmatrix} 1 & 2 & 1 & k \\ 2 & 1 & 4 & 6 \\ 1 & -4 & 5 & 9 \end{pmatrix}$	2 Total = 2
(ii)	$\begin{pmatrix} 1 & 2 & 1 & k \\ 0 & 3 & -2 & 2k-6 \\ 0 & 6 & -4 & k-9 \end{pmatrix}$ $2R_1 - R_2$ $\begin{pmatrix} 1 & 2 & 1 & k \\ 0 & 3 & -2 & 2k-6 \\ 0 & 0 & 0 & 3k-3 \end{pmatrix}$	1 1 1 Total = 3
(iii)	for consistency of the system : $3k - 3 = 0$ $\Longrightarrow k = 1$	1 1 Total =2
(iv)	let $z = \lambda$ $3y - 2\lambda = -4$ $y = \frac{2\lambda - 4}{3}$ $x + 2\frac{(2\lambda-4)}{3} + \lambda = 1$ $x = \frac{11 - 7\lambda}{3}$	1 1 1 Total = 3

8	$2\begin{vmatrix} k & -1 \\ 4 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + (k)\begin{vmatrix} 1 & k \\ 3 & 4 \end{vmatrix}$ $= 0$ $-3k^2 + 8k + 3 = 0$ $3k^2 - 8k - 3 = 0$ $(3k+1)(k-3) = 0$ $k = -\frac{1}{3}$ and $k = 3$	2 [for two out of three correct] 1 1 1 1 Total =6