HARRISON COLLEGE INTERNAL EXAMINATIONS 2012 : PURE MATHEMATICS [UNIT2 – Preview TEST 3]

SOLUTIONS

1. (a) ${}^{18}C_7 = 31\,824$ [2]

   (b) number of ways of choosing a committee with 2 from St.Michael, 2 from St. Lucy and **3** from St.John

   ${}^{5}C_2 \times {}^{6}C_2 \times {}^{7}C_3 = 5250$ [2]

   So required probability = $\frac{5250}{31824}$ [1]

   (c) number of ways of choosing a committee with 3 people from St. John = ${}^{7}C_3 \times {}^{11}C_4 = 11550$ [2]

   So required probability = $\frac{11550}{31824}$ [1]

2. (i) number of different arrangements = $\frac{8!}{3! \times 2!} = 3360$ [3]

   (ii) number of arrangements with Rs together = $\frac{7!}{3!} = 840$ [2]

   (iii) Pr (R and R') + Pr(R' and R) + Pr(R and R) =

$$\left(\frac{2}{8} \times \frac{6}{7}\right) + \left(\frac{6}{8} \times \frac{2}{7}\right) + \left(\frac{2}{8} \times \frac{1}{7}\right) = \frac{26}{56} = \frac{13}{28}$$ [4]

3. (a) Since A and B are independent $\Longrightarrow P(A \cap B) = P(A) \times P(B)$

   Let P(B) = x

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   $0.9 = 0.5 + x - 0.5x$

   $x = 0.8$ [4]

   (b) Probability that A or B occurs but not both = 0.5 + 0.8 – (0.5 × 0.8) = 0.9 [2]

4. (a) $\frac{(1+2i)^2}{7-i} = \frac{(1+2i)\times(1+2i)}{7-i} \times \frac{7+i}{7+1} = \frac{-3+4i}{7-i} \times \frac{7+i}{7+i} = \frac{-25+25i}{50} = -\frac{1}{2} + \frac{1}{2}i$ [4]

(b) If $1-i$ is a root then $\Rightarrow$ $1+i$ is a root as well. [1]

The sum of the roots = -1 [1]

$(1-i) + (1+i) + x = -1$ where x is the third root. [1]

$x = -3$ [1]

5. (i) $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$ $\Rightarrow$ $= R(cos\alpha + i\ sin\alpha)$; R is the modulus and α is the principal argument

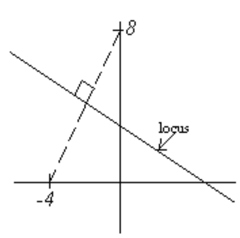
$R = \sqrt{(-\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} = 1$ [1]

$\alpha = \pi - tan^{-1}\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ [2]

So $-\frac{1}{2} + i\frac{\sqrt{3}}{2} = 1(cos\frac{2\pi}{3} + i\ sin\frac{2\pi}{3})$

(ii) $(-\frac{1}{2} + i\frac{\sqrt{3}}{2})^3 = 1^3(cos\frac{2\pi}{3} + i\ sin\frac{2\pi}{3})^3$ = $\cos 2\pi + i\ sin 2\pi$ = 1 [3]

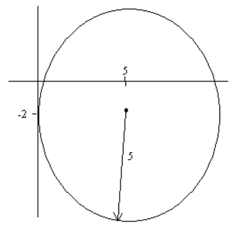
6. (i) Locus of points satisfying $|z+4| = |z-8i|$ is:
- the perpendicular bisector of the line joining [1]
- the point $(-4, 0)$ and $(0, 8)$ [1]



[1]

(ii) The locus of the points satisfying $|z - 5 + 2i| = 5$ is :

- A circle centre $(5, -2)$ [1]
- Radius = 5 units [1]



[1]

7. (i) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & -1 & -1 \\ 1 & 2 & 4 & k \end{pmatrix}$ [2]

(ii) $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & -1 & -4 & -k \end{pmatrix}$

$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1-k \end{pmatrix}$ [3]

(iii) for consistency of the system : $1 - k = 0 \implies k = 1$ [2]

(iv) let $z = \lambda$ [1]

$y + 3\lambda = 1$
$y = 1 - 3\lambda$ [1]

$x + (1 - 3\lambda) + \lambda = 0$
$x = 2\lambda - 1$ [2]

8. $1\begin{vmatrix} 0 & 2 \\ k & 6 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ -1 & 6 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 0 \\ -1 & k \end{vmatrix} = 0$ [3]

$-2k - 2(20) - 3k = 0$ [1]

$k = -8$ [1]