HARRISON COLLEGE INTERNAL EXAMINATIONS 2012 : PURE MATHEMATICS [UNIT2 – TEST: 1]

SOLUTIONS AND MARK SCHEME

Question	Working	Marks & comments
1.(a)	$log_4x + 6log_x2 = 4$	
	$log_4x + \frac{3}{log_4x} = 4$	1
	$(log_4x)^2 + 3 = 4\,log_4x$	1
	let u = log_4x	
	$u^2 + 3 = 4u$	
	$u^2 - 4u + 3 = 0$	
	$(u-1)(u-3) = 0$	1
	$\Rightarrow\ log_4x = 1 \qquad log_4x = 3$	
	$x = 4$	1
	and $x = 4^3 = 64$	1 Total = 5
(b)(i)	$V = \pi\int_0^3 x^2\ dy$	1
	$y = log_2x \Rightarrow x = 2^y$	
	$x^2 = 4^y$	1
	$= e^{yln4}$	1 Total = 3
(b)(ii)	$V = \pi\int_0^3 e^{y\,ln4}\ dy$	
	$V = \pi\,[\frac{e^{yln4}}{ln4}]_0^3$	1
	$V = \pi\left[\frac{e^{ln64}}{ln4} - \frac{1}{ln4}\right]$	1
	$V = \pi\left[\frac{63}{ln4}\right]$	Award 2 marks for $\pi\left[\frac{64}{ln4}\right]$ 1 Total =3

2	$\frac{dQ}{dt} = 5 - \frac{Q}{10}$	
	$\frac{dQ}{dt} + \frac{Q}{10} = 5$	
	Integrating factor = $e^{\int \frac{1}{10}\,dt}$	1
	$e^{\frac{t}{10}}$	1
	$\therefore\ Qe^{\frac{t}{10}} = \int 5e^{\frac{t}{10}}\ dt$	1
	$Qe^{\frac{t}{10}} = 5\frac{e^{\frac{t}{10}}}{\frac{1}{10}} + constant$	1
	$Q = 50 + A\,e^{-\frac{t}{10}}$	1
	Given t =0 Q =0	
	$A = -50 \quad \Rightarrow\ Q = 50 - 50\,e^{-\frac{t}{10}}$	1 Total = 6

3(i)	$y = (lnx)\,sin^{-1}(2x)$	
	$\frac{dy}{dx} = lnx \cdot \frac{2}{\sqrt{1-4x^2}}$	1
	+	1
	$sin^{-1}(2x) \cdot \frac{1}{x}$	1 Total =3
(ii)	$y = \frac{x}{e^{2x}}$	
	$\frac{dy}{dx} = \frac{(\qquad)}{e^{4x}}$	1
	$= \frac{e^{2x} - (\quad)}{e^{4x}}$	1
	$= \frac{e^{2x} - 2x\,e^{2x}}{e^{4x}}$	1 Total = 3

4 (a) (i)	$x = \sqrt{t} = t^{\frac{1}{2}}$ $\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$ $y = t^3 - \frac{5t^2}{2}$ $\frac{dy}{dt} = 3t^2 - 5t$ $\frac{dy}{dx} = (3t^2 - 5t) \times 2\sqrt{t}$ $= 6t^{\frac{5}{2}} - 10\, t^{\frac{3}{2}}$	 1 1 1 $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{1}{\frac{dx}{dt}}$ stated or implied 1 Total = 4
(ii)	$\frac{d^2y}{dx^2} = \left(15t^{\frac{3}{2}} - 15t^{\frac{1}{2}}\right) \times 2t^{\frac{1}{2}}$ $= 30t^2 - 30t$	1 using derivative from (i) 1 1 Total = 3
(iii)	When $t = 4$ $x = 2$ $y = 24$ $\frac{dy}{dx} = 112$ Equation of tangent: $y - 24 = 112(x - 2)$	1 1 1 using derivative from (i) 1 Total = 4
4(b)	$2\sin(xy) = 1$ Differentiating implicitly $2\cos(xy) \times \left(x\frac{dy}{dx} + y\right) = 0$ $x\frac{dy}{dx} + y = 0$ $\frac{dy}{dx} = -\frac{y}{x}$ When $y = \frac{1}{2}$ $x = \frac{5\pi}{3}$ $\frac{dy}{dx} = -\frac{3}{10\pi}$	 1 1 1 1 Total = 4

5 (i)	$f(x) = \dfrac{x^2 + 9x}{(x-1)(x^2+9)}$ $= \frac{A}{(x-1)} +$ $\frac{Bx+C}{(x^2+9)}$ $A = 1$ $B = 0$ $C = 9$	 1 1 1 1 1 Total =5
(ii)	$f(x) = \dfrac{1}{x-1} + \dfrac{9}{x^2+9}$ $\int f(x)dx = \int \dfrac{1}{x-1} + \dfrac{9}{x^2+9}\, dx$ $= ln\lvert x-1\rvert +$ $9 \times \frac{1}{3}\, tan^{-1} \frac{x}{3} + k$ $= ln\lvert x-1\rvert + 3\, tan^{-1} \frac{x}{3} + k$	 1 1 Total = 2

6 (i)	Given $\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\,dx$ Using $x = sin\theta$ $\frac{dx}{d\theta} = \cos\theta$ $x = 0 \;\; \theta = 0; \;\; x = \frac{1}{2} \;\; \theta = \frac{\pi}{6}$ Integral becomes $\int_0^{\frac{\pi}{6}} \frac{1}{(1-sin^2\theta)^{\frac{3}{2}}} \cos\theta\, d\theta$ $= \int_0^{\frac{\pi}{6}} \frac{1}{cos^2\theta}\, d\theta$ $= \int_0^{\frac{\pi}{6}} sec^2\theta\;\; d\theta$ $= \tan\theta \Big\vert_0^{\frac{\pi}{6}}$ $= \frac{1}{\sqrt{3}}$	 1 1 1 1 1 1 Total = 6
(ii)	$\int_1^3 \frac{lnx}{x^2}\, dx$ $= -\frac{1}{x}\, lnx$ $+ \int \frac{1}{x^2}\, dx$ $= -\frac{1}{x}\, lnx - \frac{1}{x} \Big\vert_1^3$ $= \left(-\frac{1}{3}\, ln3 - \frac{1}{3}\right) - (0 - 1)$ $= \frac{2}{3} - \frac{1}{3}\, ln3$	 1 1 1 1 1 Total = 5

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x	$\sqrt{x^2 + 4}$	
0	2	
2		2.828
4		4.472
6	6.324	

Area = $\frac{1}{2} \times 2 \times \left(8.324 + 2 \times (7.300)\right)$

= 22.924

= 22.9

1 for using 4 ordinates

1 evaluating $f(x)'s$

1 for correct use of trapezium Rule

1

Total = 4