HARRISON COLLEGE INTERNAL EXAMINATION 2012
CARIBBEAN ADVANCED PROFICIENY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 – TEST 2 (Preview)
TIME: 1 hour 30 minutes

This examination paper consists of 2 printed pages.
This paper consists of 10 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

Examination materials allowed

1. Mathematical Formulae
2. Scientific calculator (non-programmable, not graphical)

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1) i) List the first five terms of the sequence with general term $u_n = \frac{n^2-1}{n+1}$. [2]

   ii) Is the sequence convergent or divergent? Give a concise reason for your answer. [2]

   Total 4 marks

2) Evaluate the following without writing out the series in full$\sum_0^{50}(25-2r)$. [3]

   Total 3 marks

3) A student reading a 426-page book finds that he reads faster as he better understands ~~gets into~~ the subject. He reads 19 pages on the first day, and his rate of reading goes up by 3 pages each day. How long does he take to finish the book? [6]

   Total 6 marks

4) A student negotiated a deal with his parents as a reward for excellent Common Entrance results. He is to receive \$1 on 1 July, \$4 on 1 August, \$16 on 1 September, and so on until the last payment on 1 December. How much money does the student receive in total? [4]

   Total 4 marks

5) Find and simplify the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^9$. [5]

Total 5 marks

6) i) Expand $(1 + 2x)^{\frac{1}{2}}$ as a series of ascending powers of $x$as far as the term in x^3. [3]

ii) By substituting $x = \frac{1}{25}$ in your expansion from i), find an approximate value for $\sqrt{3}$, giving your answer correct to three (3) decimal places. [4]

Total 7 marks

7) i) Show that a root, α, of the equation $x^3 - 7x - 12 = 0$ lies between 3 and 4. [2]

ii) Using the Newton Raphson formula, show that if x_n is the n^th approximation to α, then

$$x_{n+1} = \frac{2x_n^3 + 12}{3x_n^2 - 7}$$ [3]

iii) Hence, using $x_0 = 3.2$ as a first approximation to the real root α, find this root correct to 2 decimal places. [5]

Total 10 marks

8) Prove by the method of mathematical induction that for all positive integers n, ~~$8^n + 6$ is divisible by 14.~~
Sequence or Series instead (but can leave in for this Preview) [7]

Total 7 marks

9) Show that $r(r+1)(r+2) - (r-1)r(r+1) \equiv 3r(r+1)$.

Hence find the sum of the series $\sum_{r=1}^{n} r(r+1)$. [6]

Total 6 marks

10) Use the results of the Maclaurin's series expansions of $ln(1+x)$ and $sinx$ to expand $ln(1+sinx)$ in ascending powers of x as far as the term in x^4. [8]

Total 8 marks

END OF TEST

2012 Unit 2 –Module 2 Preview Test Solutions

1. $u_n = \frac{n^2-1}{n+1}$

First 5 terms in sequence are $\frac{1^2-1}{1+1}, \frac{2^2-1}{2+1}, \frac{3^2-1}{3+1}, \frac{4^2-1}{4+1}, \frac{5^2-1}{5+1},$

$= 0, 1, 2, 3, 4, \ldots$

$$\lim_{n\to\infty} \frac{n^2-1}{n+1} = \lim_{n\to\infty} \frac{\cancel{(n+1)}(n-1)}{\cancel{n+1}} = \lim_{n\to\infty} n-1 = \infty$$

Since there is no limit as n tends to infinity, the sequence is divergent.

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2. $\sum_0^{50}(25-2r) = 25-2(0) + \{25-2(1)\} + \{25-2(2)\} + \{25-2(3)\} + \cdots \{25-2(50)\}$

$$= (25 \times 51) - 0 - 2 - 4 - 6 - \cdots - 100$$

$$= 1275 - \left(\frac{50}{2}(4 + 49(2))\right)$$

$$1275 - 2550 = -1275$$

OR $\sum_0^{50}(25-2r) = \sum_0^{50} 25 - 2\sum_0^{50} r$

= (51 × 25) – 2[sum of an AP, with $a = 0$, $d = 1$ & $n = 51$]

= 1275 – 2[2550]

= - 1275

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3. A.P. with $a = 19$ *and* $d = 3$ *and* $S_n = 426$

$$426 = \frac{n}{2}\{2(19) + (n-1)3\}$$

$$852 = n\{38 + 3n - 3\}$$

$$852 = 35n + 3n^2$$

$$3n^2 + 35n - 852 = 0$$

$$n = \frac{-35 \pm \sqrt{35^2 - 4(3)(-852)}}{6}$$

$$n = \frac{-35 \pm 107}{6} = 12$$

He takes 12 days to finish the book.

N.B. We only take the positive value of n.

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4. $a = 1, r = 4, n = 6$

$S_6 = 1\frac{(4^6-1)}{4-1} = \$1{,}365.$

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5. $\left(2x + \frac{1}{x^2}\right)^9 = (2x)^9 + \binom{9}{1}(2x)^8\left(\frac{1}{x^2}\right)^1 + \binom{9}{2}(2x)^7\left(\frac{1}{x^2}\right)^2 + \binom{9}{3}(2x)^6\left(\frac{1}{x^2}\right)^3 + \ldots$

Term independent of x is $\binom{9}{3}(2)^6 = 84 \times 64 = 5376$

OR General term is of the form $\underset{r}{\overset{n}{C}}(2x)^{n-r}(\frac{1}{x^2})^r$

To find r; Now $(x)^{n-r}\ (x^{-2})^r \equiv x^0$ i.e. $(x)^{9-r}\ (x^{-2})^r \equiv x^0$
$(x)^{9-r-2r} \equiv x^0 \rightarrow r = 3$

Term independent of x is $\underset{3}{\overset{9}{C}}(2x)^{9-3}(\frac{1}{x^2})^3 = 84 \times (2)^6 \times (\frac{1}{1})^3 = 5376$

6.i) $(1+2x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)(2x) + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)}{2!}(2x)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)}{3!}(2x)^3 + \cdots$

$$= 1 + x - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \cdots$$

ii) $\left[1 + 2\left(\frac{1}{25}\right)\right]^{\frac{1}{2}} = \left(\frac{27}{25}\right)^{\frac{1}{2}} = \sqrt{\frac{9\times 3}{25}} = \frac{3}{5}\sqrt{3}$

So $1 + \frac{1}{25} - \left(\frac{1}{2}\right)\left(\frac{1}{25}\right)^2 + \left(\frac{1}{2}\right)\left(\frac{1}{25}\right)^3 \approx \frac{3}{5}\sqrt{3}$

$$\frac{5}{3}\left(1 + \frac{1}{25} - \frac{1}{1250}\right) \approx \sqrt{3}$$

$$\sqrt{3} \approx 1.732 \textit{ to 3 decimal places}$$

7. i) $f(x) = x^3 - 7x - 12$

$$f(3) = 27 - 7(3) - 12 = -6$$

$$f(4) = 4^3 - 7(4) - 12 = 24$$

There is a sign change between $x = 3$ *and* $x = 4$ AND $f(x)$ is continuous between $x = 3$ and $x = 4$, so α lies between 3 and 4.

ii) $f(x) = x^3 - 7x - 12$

$$f'(x) = 3x^2 - 7$$

$$x_{n+1} = x_n - \frac{x_n^3 - 7x_n - 12}{3x_n^2 - 7}$$

$$= \frac{x_n(3x_n^2 - 7) - (x_n^3 - 7x_n - 12)}{3x_n^2 - 7}$$

$$= \frac{3x_n^3 - \cancel{7x_n} - x_n^3 + \cancel{7x_n} + 12}{3x_n^2 - 7} = \frac{2x_n^3 + 12}{3x_n^2 - 7}\ Q.E.D.$$

iii) $x_0 = 3.2$

$$x_1 = \frac{2(3.2)^3 + 12}{3(3.2)^2 - 7} = 3.269$$

$$x_2 = \frac{2(3.269)^3 + 12}{3(3.269)^2 - 7} = 3.267$$

Hence $x = 3.27$ *to 2 decimal places*

8. $Let\ P_n\ be\ 8^n + 6$

Let $n = 1\ so\ 8^1 + 6 = 14$ which is divisible by 14 so P_1 is true

Assume that $P_k is\ true, i.e\ that\ 8^k + 6\ is\ divisible\ by\ 14$

To Show that $P_{k+1} - P_k$ is divisible by 14

$$8^{k+1} + 6 - (8^k + 6) = 8^k 8^1 + \cancel{6} - 8^k - \cancel{6}$$

$$= 8^k(8-1) = 7(8^k) = \frac{14}{2}(2^3)^k$$

$P_{k+1} - P_k = 14 \times \frac{2^{3k}}{2} = 14 \times 2^{3k-1}$ which is divisible by 14

So $P_{k+1} = P_k + 14 \times 2^{3k-1}$ is also divisible by 14,

Hence by induction, P_n is true for $n = 1, 2, 3, \ldots$

9. $r(r+1)(r+2) - (r-1)r(r+1)$

$$= r(r+1)[r+2-(r-1)] = r(r+1)3$$

$$= 3r(r+1) \quad Q.E.D$$

$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}\sum_{r=1}^{n} 3r(r+1)$

Using the Method of Differences

$$= \frac{1}{3}\{\cancel{1(2)(3)} - 0 + \cancel{2(3)(4)} - \cancel{(1)(2)(3)} + \cancel{3(4)(5)} - \cancel{(2)(3)(4)} + \ldots + \cancel{(n-1)(n)(n+1)} - \cancel{(n-2)(n-1)(n)} + n(n+1)(n+2) - \cancel{(n-1)(n)(n+1)}\}$$

$$= \frac{1}{3}n(n+1)(n+2)$$

10. $ln(1+X) = X - \frac{X^2}{2} + \frac{X^3}{3} - \frac{X^4}{4} + \cdots$

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots = x - \frac{x^3}{6} + \cdots$$

Let $sinx = X$

$$ln(1 + sinx) = sinx - \frac{(sinx)^2}{2} + \frac{(sinx)^3}{3} - \frac{(sinx)^4}{4} + \cdots$$

$$= \left(x - \frac{x^3}{6}\right) - \frac{1}{2}\left(x - \frac{x^3}{6}\right)^2 + \frac{1}{3}\left(x - \frac{x^3}{6}\right)^3 - \frac{1}{4}\left(x - \frac{x^3}{6}\right)^4 + \cdots$$

$$= x - \frac{x^3}{6} - \frac{1}{2}\left(x^2 - \frac{x^4}{3} + \cdots\right) + \frac{1}{3}(x^3 - \cdots) - \frac{1}{4}(x^4 - \cdots) + \cdots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots$$

THE END