HARRISON COLLEGE INTERNAL EXAMINATION 2012
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

SCHOOL BASED ASSESSMENT

PURE MATHEMATICS
UNIT 2 - TEST 1

TIME: 1 Hour & 30 minutes

This examination paper consists of 2 printed pages.
The paper consists of 7 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Number your questions carefully and do **NOT** write your solutions to different questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae
2. Electronic calculator (non-programmable, non-graphical)

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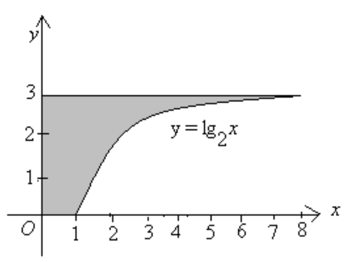
1. (a) Solve $log_4 x + 6log_x 2 = 4$, [5]

(b)



The diagram above (not drawn to scale) shows the graph of $y = log_2 x$ between *x* = 1 and *x* = 8. The shaded region, bounded by $y = log_2 x$, the line *y* = 3, and the *x* and *y* axes, is rotated about the *y*-axis to form a solid.

(i) Show that the volume of the solid formed is given by $V = \pi \int_0^3 e^{y\, ln4}\ dy$. [3]

(ii) Hence find the value for the volume of the solid in the form $\frac{a}{\ln b}\pi$ where *a* and *b* are real numbers. [3]

2. The Barbados Water Authority has a reservoir which is to be partly refilled by water supplied by a desalination plant. The desalinated water contains a small quantity of salt and it has been determined that the differential equation

$$\frac{dQ}{dt} = 5 - \frac{Q}{10}$$

where Q is the quantity of salt, in kg, in the reservoir on any day and *t* is the time in days, describes the rate of change of salt in the reservoir.

Assume there was no salt in the reservoir originally; solve this differential equation to find Q, the quantity of salt in the reservoir at any time *t*. [6]

3. Find $\frac{dy}{dx}$ when:

(i) $y = (lnx)\, sin^{-1}(2x)$ [3]

(ii) $y = \frac{x}{e^{2x}}$ [3]

4. (a) Given $x = \sqrt{t}$ and $y = t^3 - \frac{5t^2}{2}$ for $t > 0$

(i) Obtain a simplified expression in terms of *t* for $\frac{dy}{dx}$. [4]

(ii) Show that $\frac{d^2y}{dx^2} = at^2 + bt$, determining the values of the constants *a* and *b*. [3]

(iii) Find the equation of the tangent to the curve at the point where $t = 4$. [4]

(b) Find the gradient of the curve given by $2\sin(xy) = 1$, when $y = \frac{1}{2}$ and $x = \frac{5\pi}{3}$. [4]

5. It is given that $f(x) = \frac{x^2 + 9x}{(x-1)(x^2+9)}$

(i) Express *f(x)* in partial fractions. [5]

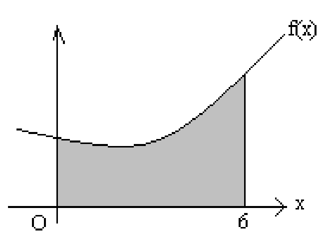
(ii) Hence find $\int f(x)dx$ [2]

6. (i) Use the substitution $x = \sin\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{1}{(1-x^2)^{\frac{3}{2}}}\, dx$$ [6]

(ii) Using integration by parts, find the exact value of $\int_1^3 \frac{lnx}{x^2}\, dx$. [5]

7.



The shaded region in the diagram above is bounded by the curve $f(x) = \sqrt{x^2 + 4}$, the x-axis and the lines $x = 0$ and $x = 6$.

Use the trapezium rule with three intervals of equal width to estimate the area of the shaded region. [4]