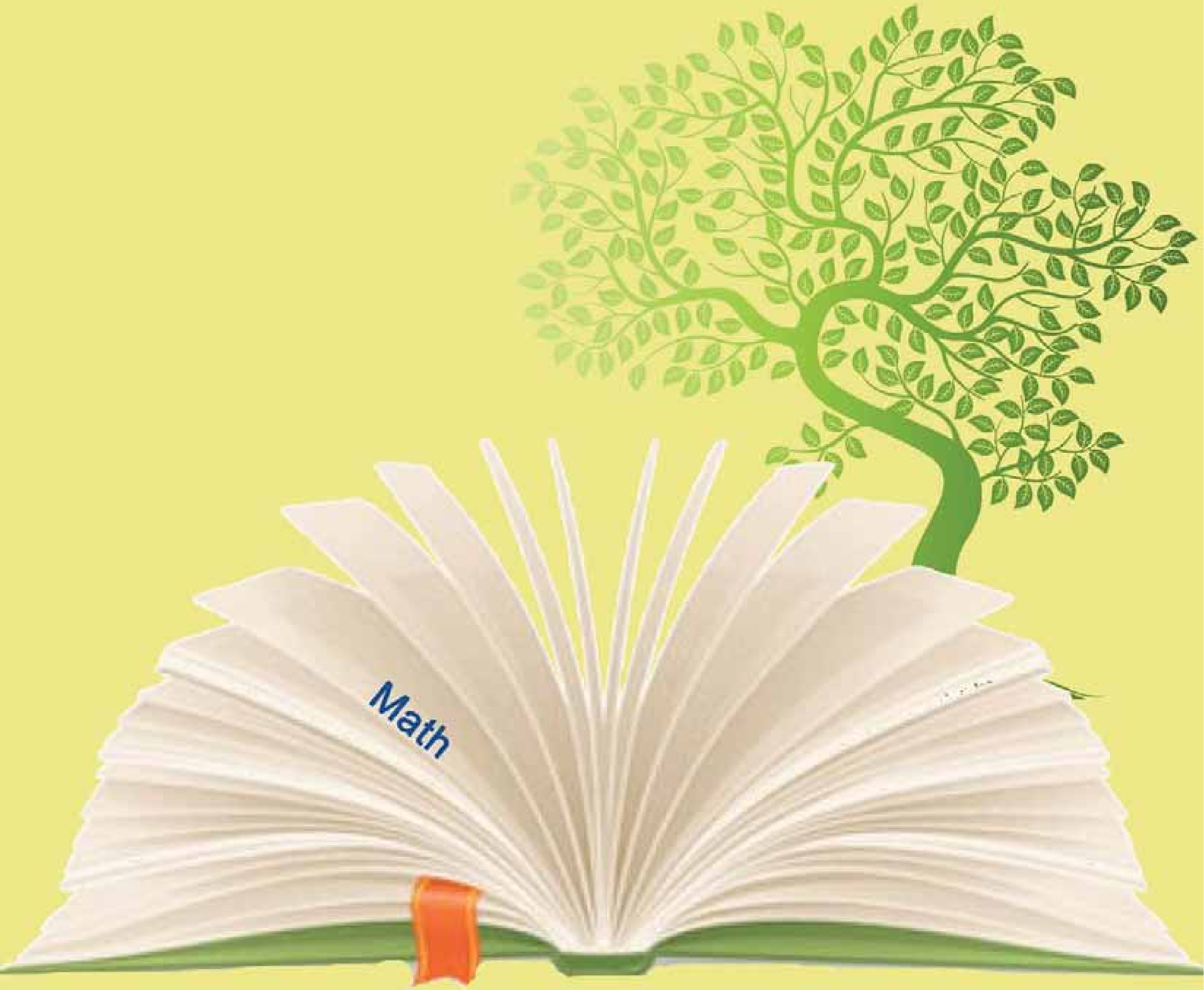


OPEN EDUCATIONAL RESOURCES 4 OPEN SCHOOLS

Taking Education to the People

Unit 13

Ratio and Proportion

Introduction

While studying mathematics at the junior secondary level, you learned how to:

- compare like quantities by ratios,
- find missing quantities in a ratio, and
- compare unlike quantities by rates.

You also did some work on direct and indirect proportion. Proportion is used with quantities that are seemingly unrelated.

In this unit, we are going to continue work on ratio, rates, and proportion. Have you ever thought of why ratios, proportions, and rates are important?

Let us look at the following situations:

- If you have ever travelled by public transport, you have definitely paid the bus or taxi fare.
- If you have ever bought groceries for your family, you have definitely paid the money for each item you buy.
- If you have attended a party or feast and helped in the preparations, especially food preparations for the guests, you probably diluted some kind of concentrated drink.
- If you have ever been involved in building a house, you will definitely need the knowledge from this unit.

These are some examples where the knowledge of how to use ratios, proportions, and rates can help you make informed decisions in everyday life especially in commercial math (unit 9).

This unit consists of 45 pages. This is 1% of the whole course, so plan your time accordingly. As reference, you will need to devote 15 hours to work on this unit, 10 hours for formal study and 5 hours for self-study and completing assessments/assignments.

This Unit is Comprised of Three Lessons:

Lesson 1 Ratios
Lesson 2 Rate
Lesson 3 Proportion

Upon completion of this unit you will be able to:

Outcomes

- *divide* a quantity in a given ratio;
- *increase and decrease* a quantity by a given ratio;
- *calculate* average speed;
- *formulate* equations from direct and inverse proportion situations, and use these equations to solve for one quantity if given others;
- *apply* the ideas and notation of ratio, proportion, and rate to practical situations.

Terminology

Ratio: The comparison of two or more like quantities, that is, quantities measured in the same units.

Rate: A special type of ratio that is used when comparing quantities measured in different units.

Proportion: Statement of equality between two ratios or rates.

Online Resource

If you can get on the internet please utilize the resources at www.hippocampus.org. It is an excellent source of information for mathematics and the topics discussed in this unit. Here you will find:

- Presentations
- Simulations
- Videos
- Online Study Groups
- Links to Even More Information
- Textbook Correlations
- Online Courses

Lesson 1 Ratios

In Lesotho, children are often given sweets or another treat when visitors come to their homes or when they accompany their elders to visit another household. Sometimes, the sweets must be shared among a number of children. When I was growing up, we had to share everything, usually with the biggest portion going to the youngest and so on all the way up to the eldest. Often, the eldest child got only a small share or nothing at all.

I also felt this was terribly unfair, except while I was the youngest. But at the time, I didn't understand mathematics. The fairest and most common way of sharing would be to use a ratio to divide the sweets. How do we do this? Keep your eyes open for the answer down below.

At the end of this subunit you should be able to:

- define the term ‘ratio’;
- write a ratio in fraction form;
- present examples of how ratios are used in everyday life;
- divide a quantity in a given ratio.

There are 6 pages on this sub-unit on ratios.

Simple Ratios

Do you remember how to do fractions from your previous studies? If not, this example might help to revise these concepts.

Imagine that you have gone to your neighbour’s house and a peach pie is brought out to share among everyone there. One family has four members, the other family has three members. Each member is to get an equal share.

The **first step** here is to find how much is going to be shared

One pie

The **second step** is to find how many shares are needed. The total number of people who are going to be sharing the pie is the sum of the members from the two families:

$4 + 3 = 7$

So, we need to cut the pie into seven equal slices so that each person gets an equal share. We can say that the pie has been cut up in proportion to the number of people who will be sharing it. If we compare the slice that each person gets to the whole pie, this can be shown as:

$\frac{1}{7}$, which can also be written as 1 : 7.

1 is the first term of the ratio and 7 is the second term of the ratio.

Other relationships could also be expressed as ratios. For example, if we had a bag full of pies of different flavours, we could show the ratio between one flavour and another using a ratio.

A ratio expresses a relationship between two or more like quantities.

When we say ‘like quantities’, we mean things that are alike in some way. In order to have a ratio, the quantities of the two things must be expressed in the same unit of measurement. If you need to revise this concept, go back to Unit 1 of this course and read through the materials again.

Example

Bohlokoa has three apples and two oranges; these can't be compared directly with each other in a ratio because they are different things. Apples are not exactly the same as oranges – they are not 'like quantities'.

However, if we think of them as two different types of the same thing – fruit – then a comparison can be made in the form of a ratio.

3 apples (fruit) compared to 2 oranges (fruit)

Which can also be written as $\frac{3}{2}$ or 3: 2

In our everyday life we make a lot of comparisons. If there are 16 orange trees and 12 guava trees in an orchard, then the ratio of orange trees to guava trees is:

$$\frac{16}{12}$$

However, ratios need to be reduced to their simplest form. To do this you use the same process that you learned for simplifying fractions. You look for common factors and divide the terms both above and below the line by the same factor.

In this example, the figures both above and below the line can be divided by a factor of four. So, when this operation is complete our ratio looks like this in its simplest form:

$\frac{4}{3}$, which can also be written 4 : 3

Ratios with more than two parts

Let's go back to the example of building a house, which usually involves mixing cement, sand and gravel to make concrete. However, depending on what the concrete will be used for, the amount of each ingredient can be varied in the mix. If you look at the instructions on the back of a pocket or bag of cement you will see a table of numbers similar to the one below (*the numbers may differ from one brand of cement to another):*

Purpose of mix	**Cement**	**Sand**	**Gravel**
Home concrete	1	$3\frac{1}{2}$	$3\frac{1}{2}$
Medium strength	1	$2\frac{1}{2}$	$2\frac{1}{2}$
Reinforced concrete	1	2	2

Watertight concrete	1	$1\frac{1}{2}$	$1\frac{1}{2}$
Mortar for bricklaying and plastering	1	6	-
Brick and block mix	1	8	-

The numbers inside the table show the amounts of each material used to make concrete for different purposes. The numbers have been compared to help the builder mix the concrete correctly. If you use a shovel to measure, then you will use one shovel of cement and six shovels of sand to make mortar.

If you use a wheelbarrow to measure, you will use one wheelbarrow full of cement and six wheelbarrows of sand to make mortar. Notice that, when we say ‘like quantities’, we mean the same units of measurement; if you use a shovel to measure one ingredient, you must use a shovel for all the others. It would be the same if you use a wheelbarrow, a bucket or anything else to measure the different materials in your mix. However, no matter what measure you use, the ratio between the different materials stays the same.

For home concrete, the mix requires a different ratio; it should have one measure of cement, three and a half measures of sand and three and a half measures of gravel. This can be expressed as a ratio between the three ingredients as follows:

$$1: 3\frac{1}{2} : 3\frac{1}{2}$$

The ratio of cement to gravel in our home concrete mix is $1: 3\frac{1}{2}$

However, if we want to calculate the ratio of cement to total mixture, we need to carry out a process of addition to find the total amount in the mix:

The total number of ‘like quantities’ (i.e. parts, shovels, wheelbarrows, buckets, etc.) in the mixture is the sum of all the quantities:

$$1 + 3\frac{1}{2} + 3\frac{1}{2} = 8$$

Since only one measure of cement is needed, the ratio of cement to the total mixture is

1: 8 which can also be written as $\frac{1}{8}$ in fraction form.

Now, try writing the ratio for medium strength concrete mix:

The ratio for medium strength concrete mix is $1 : 2\frac{1}{2} : 2\frac{1}{2}$

What is the ratio of cement to sand?

The ratio of cement to sand $1 : 2\frac{1}{2}$

What is the ratio of cement to the total amount in the mix?

The total number of like quantities is

$1 + 2\frac{1}{2} + 2\frac{1}{2} = 6$

The ratio of cement to the total amount in the mix is 1 : 6 which can also be written as $\frac{1}{6}$ in fraction form.

Activity

Activity 13.1

1. A box contains 20 red balls and 30 black balls. Write down the ratio of the red balls to the black balls.

2. A family's income is M8 000 per month, which is spent as follows:

Accommodation (Rent)	M 1 500
Food	M 2 400
Utilities	M 500
Education Expenses	M 1 200
Transport	M 800
Savings & Investment	M 400
Debt Repayments	M 500
Other Expenses	M 700

Write as ratios in their simplest forms:

a) the ratio of the amount spent on grocery to the total income;

b) the ratio of the money spent on transport to rent;

c) the ratio of money spent on savings to other expenses;

d) the ratio of money spent on school fees to rent.

Compare your answers to those given at the end of the sub-unit. Note that it is important to understand this concept. If you do not understand it, review the above content and try the activity again.

Model Answers

Activity 13.1

1. 20 : 30 *(divide by 10)*

   2 : 3 or $\frac{2}{3}$

2.

   a. 2400 : 8000 *(divide by 800)*

      3 : 10 or $\frac{3}{10}$

   b. 800 : 1500 *(divide by100)*

      8: 15 or $\frac{8}{15}$

   c. 400 : 700 *(divide by 100)*

      4 : 7 or $\frac{4}{7}$

   d. 1200 : 1500 (divide by 300)

      4 : 5 or $\frac{4}{5}$

Dividing a quantity in a given ratio

You have just done some work on writing ratios and simplifying them where possible. These two skills will enable you to use a ratio to divide a given quantity in a ratio.

Consider the following case.

Example 1

A basket contains brown eggs and white eggs in a ratio of 3 : 4. If there are 42 eggs, find the number of eggs of each colour.

Solution

The ratio $3:4$ has already been reduced to its simplest form. In order to find out the total number of 'like measures' there are in this example, we add the two terms of the ratio together, as follows:

$$3 + 4 = 7$$

Therefore the ratio of brown eggs to the total is $\frac{3}{7}$.

To find the number of brown eggs in the basket, we multiply the ratio by the total number of eggs:

$$= \frac{3}{7} \times 42$$

$= 18$ brown eggs

The ratio of white eggs to the total is $\frac{4}{7}$.

Using the same process as we used above, the number of white eggs

$$= \frac{4}{7} \times 42$$

$= 24$ white eggs

We can check our answers by adding them together to see whether they equal the total number of eggs in the basket.

Tip

$$18 + 24 = 42$$

Alternative way of working

There is another way of solving this problem, which you might find easier to understand. Run through the following steps for the example above and see whether if makes more sense to do it this way.

STEP 1 Simplify the ratio, if this is possible. (This may not always be required, but calculations are much easier to perform when the ratio is in its simple form.)

$3 : 4$ is already in simple form

STEP 2 Add the numbers in the ratio together to get total number of shares or units of 'like measurement':

$3 + 4 = 7$

STEP 3 Divide the total number of items (in these case, eggs) by the total number of shares (from Step 2 above) to get the amount for a single share.

$\frac{42}{7} = 6$ (this means one share is equal to 6)

STEP 4 Multiply each of the numbers in the ratio by the answer in Step 3

$3 \times 6 = 18$

The number of brown eggs is 18.

4 x 6 = 24

The number of white eggs is 24.

Example 2

The ratio between the sizes of the three angles of a triangle is $3 : 2 : 1$

Calculate the size of each angle of the triangle.

Solution

Let's start with the alternative method for solving problems such as this. In order to find the number of degrees represented by each of the terms or numbers in the ratio:

STEP 1 Write ratio in simplest form, if possible. Here it is already in simplest form:

3 : 2: 1

STEP 2 Add the numbers in the ratio together. This gives $3 + 2 + 1 = 6$. This is the total number of shares or units of 'like measurement' in the ratio.

STEP 3 Now, we need the total number of degrees you get when you add up the internal angles of a triangle. Can you remember how many degrees that is?

I hope you remembered that the sum of the angles in a triangle is 180°. Now, you need to divide the 180° by 6 to get the number of degrees in one share.

$$\frac{180^\circ}{6} = 30^\circ$$

STEP 4 Multiply the amount for each share by each of the terms or numbers in the ratio to get the number of degrees for each angle.

The number of degrees for 3 shares is:

$$30^\circ \times 3 = 90^\circ$$

The number of degrees in the angle with 2 shares is:

$$30^\circ \times 2 = 60^\circ$$

The number of degrees for the angle with a single share is:

$$30^\circ \times 1 = 30^\circ$$

Check our solution by adding the answers for the three angels:

$90^0 + 60^0 + 30^0 = 180^0$

Look at the following set of equations which show another way of working out the number of degrees using the same ratio as in the example above. Why do they give the same result?

Angle A $= \frac{3}{6} \times 180^\circ = 90^\circ$

Angle B $= \frac{2}{6} \times 180^\circ = 60^\circ$

Angle C $= \frac{1}{6} \times 180^\circ = 30^\circ$

Example 3

The ratio of a man's mass to that of his wife is $7 : 5$

What is the man's mass if his wife has a mass of 75kg?

Solution

The mass of the man is calculated as follows:

STEP 1 Write the ratio in its simplest form, if possible. (Here it is already in simplest form.)

7 : 5

STEP 2 There is no need to find the total of the terms or numbers in the ratio as we can get the amount for each share by using the wife's mass and the number in the ratio for the wife's mass. (step 3)

STEP 3 Divide the $75\ kg$ by 5 to get the amount representing one share.

$$\frac{75\ kg}{5} = 15\ kg$$

STEP 4 Multiply the amount for each share by the term or number in the ratio representing the mass' mass.

$$15 \times 7 = 105\ kg$$

The solution can also be worked out as follows. Why does it give same result?

$$\frac{5}{12} \times total\ mass = 75kg$$

Simplify the above equation by making the total mass the subject:

$$total\ mass = 75kg \times \frac{12}{5}$$

$$= 180kg$$

The man's mass = *total mass – wife's mass*

$$= 180kg - 75kg$$

$$= 105kg$$

Note it

A ratio expresses a relationship between two or more like quantities.

When there are only two quantities being compared, the ratio m to n can be written in the form $m : n$ or $\frac{m}{n}$

When more than two quantities are compared, the ratio should be written as $m : n : o : p$ (and so forth).

A ratio is usually expressed in its simplest form.

Activity 13.2

1. Thabiso was born in 1950 and Clarke was born in 1960. In his will, their father said that a sum of M11 700 should be divided between them in the ratio of their ages at the time of his death. If their father died in 2005, how much did each son receive?

2. Divide 81 sweets between three children in the ratio $2 : 3 : 4$

3. The ratio of Tumo's money to Manti's money is $5 : 4$. If Tumo has Pula 45, how much has Manti?

Model Answers

Activity 13.2

1.

STEP 1 Find the ratio between Thabiso's age and Clarke's age at the time of their father's death.

Thabiso'age: Clarke's age

$(2005 - 1950) : (2005 - 1960)$

$55 : 45$

$11:9$ (simplest form)

STEP 2 Add the terms or numbers in the ratio together to find the total of shares:

$11 + 9 = 20$

STEP 3 Divide the sum of money by the total to get the size of each share

$$\frac{M11\ 700}{20} = M585$$

STEP 4 Multiply the size of a single share by the number in the ratio representing each son:

Thabiso's amount $= M585 \times 11 = M6435$

Clarke's amount $= M585 \times 9 = M5265$

Have a look at the following working. Why does it give the same result?

Thabiso received $\frac{55}{100} \times M11700 = M6435$

Clarke received $\frac{45}{100} \times M11700 = M5265$

2.

STEP 1 Write the ratio in its simplest form, if possible:

2:3:4 (already in simplest form)

STEP 2 Add the terms or numbers in the ratio to get the total number of shares:

$2 + 3 + 4 = 9$

STEP 3 Divide the total number of sweets by the total of ratio numbers to get the number in one share:

$$\frac{81}{9} = 9$$

STEP 4 Multiply each of the terms or numbers in the ratio to get the share for each child.

For the child with 2 shares:

$2 \times 9 = 18\ sweets$

For the child with 3 shares:

$3 \times 9 = 27\ sweets$

For the child with 4 shares:

$4 \times 9 = 36\ sweets$

Check you solution by adding up all the answers:

18 + 27 +36 = 81 sweets

Once more, can you suggest why the following working gives the same result?

$81\ sweets \times \frac{2}{9} = 18\ sweets$

$81\ sweets \times \frac{3}{9} = 27 sweets$

$81\ sweets \times \frac{4}{9} = 36\ sweets$

3. Remember the ratio of mass of a man to mass of his wife. This is a similar problem where you are already given the fact that Tumi has 5 shares and are asked to find Manti's shares.

STEP 1 Write the ratio in simplest form, if possible:

Tumi's money: Manti's money

$5:4$ (already in its simplest form)

STEP 2 There is no need to find total of the terms or numbers in the ration. You can get it from Tumi's money. (Step 3)

STEP 3 To find the amount for one share, use the amount of money Tumi has in his five shares:

$\frac{P45}{5} = P9$

STEP 4 To find how much money Manti has, multiply the amount for one share by the number representing Manti's money in the ratio:

$P9 \times 4 = P36$

Compare the answers below with the ones found above. Can you explain why you get the same result even though you went about it a different way?

Tumo's money is calculated as follows:

$$\frac{5}{9} \times total\ money = P\ 45$$

Therefore:

$$total\ money = P\ 45 \times \frac{9}{5}$$

$$= P\ 81$$

Manti has $P\ 81 - P\ 45 = P\ 56$

How would you go about checking your solution to this problem?

Key Points to Remember

The key points to remember in this subunit on ratios are:

- Simplify the ratio, if possible, by dividing the terms or numbers both about and below the line by the same factor.
- Add the numbers in the ratio together to get the total number of shares.
- Divide the given quantity by the total number of shares in the ratio. (Or write each of the terms/numbers in the ratio as a fraction of the total).
- Multiply the answer from 3 above by each term/number in the ratio. (Or multiply the given quantity by the fraction from 3 above for all the terms/numbers in the ratio).

You have now completed the subunit on ratios. Do a quick review of the content of this subunit and then continue on to the next subunit of Rate.

Lesson 2 Rate

Ratios compare two or more like quantities. In other words, the quantities must be expressed in the same units in order to forma ratio. But, what can we use when we need to relate two things which have different units.

Recall the example of travelling by public transportation that I raised at the beginning of this unit. The bus or taxi fare you have to pay is related to the distance you travel. There is no question that distance and money are measured in different units. For this kind of relationship, a special word is used; that word is **rate**. Can you think of other examples of rates used when talking about transport, travel and vehicles?

How much do you have to pay for transporting goods in a hired van? How much do you have to pay for a litre of petrol or diesel? How many kilometres does your vehicle travel in one hour?

The answers to all these questions must be stated in terms of two quantities with different units of measurement, either Maloti per kilometre, Maloti per litre or kilometres per hour. These are all examples of the mathematical expression called rate.

At the end of this subunit you should be able to:

- *define* the term 'rate.'
- *change* the units of measurement to equivalent units.
- *calculate* average speed.

There are 3 pages in this subunit.

Meaning of the term 'rate'

We talk about ratio when the comparison or relationship is between two or more quantities which are alike. However, there are cases where we do compare two or more quantities which are completely different, as in the comparison between the quantity of paraffin you get and the price you pay for it. If you buy two litres, you get two times the amount of paraffin and you also pay twice as much money.

In the introduction to this subunit, I introduced the term **rate** to express a relationship between two or more unlike quantities or things measured in different units.

There are some similarities between ratios and rates. First, rates can be written in a form very similar to what we have been using to write ratios. For example, if a car travels sixty kilometres in one hour, this can be written as:

$$60\ km\ per\ hour = \frac{60\ km}{1\ hour}$$

Notice that $\frac{60\ km}{1\ hour}$ is in the form of a fraction, but that the units above and below the line are different. This form is useful during calculations when you have to work out the units for the calculated answers. Let us move to equivalent rates.

When we talk about equivalent rates, using the same example above, you can write

$$\frac{60\ km}{1\ hour} = \frac{120\ km}{2\ hours} = \frac{180\ km}{3\ hours}$$

All of the above expressions are equivalent – you can replace one of them with any of the others without changing the units of measurement. All of them are expressed in kilometres per hour.

Changing measurement units to equivalent units

Another similarity between ratios and rates is the ability to change the units of measurement by using conversation factors. What does changing measurement units mean? Let's return to the example above.

If you want to express the above speed in kilometres per minute, you can substitute 60 minutes for 1 hour. The rate will then look like this:

$$\frac{60\,kilometres}{60\min utes}$$

You then need to divide both the top and the bottom of this fraction by a factor of 60 to reduce it to its simplest form:

$$\frac{1\,kilometre}{1\min ute}$$, or one kilometre per minute.

Suppose, you want to change the distance units from kilometres to metres and the time units from hours to seconds. How do you do this?

Remember, how you convert kilometres to metres and hours to seconds? The same knowledge is used here.

1 km = 1000 m
1 hour = 3600 seconds

$$\frac{60\ km}{1\ hour} = \frac{60 \times 1\ km}{1 \times 1\ hour} = \frac{60 \times 1000\ m}{1 \times 3600\ s} = \frac{60\ 000\ m}{3\ 600\ s}$$

$$= \frac{600\ m}{36\ s} = \frac{16\frac{24}{36}\ m}{1\ s} = \frac{16\frac{2}{3}\ m}{1\ s} = 16\frac{2}{3}\ m/s$$

So, 60 kilometres per hour is the same as one kilometre per minute or $16\frac{2}{3}$ metres per second. The rate remains the same, only the units of measurement change.

Activity

Activity 13.3

This activity serves as a reminder to what you have learned in your Mathematics course at junior secondary level. Therefore, in this activity you are going to find average speed, distance travelled, and time taken to complete a journey.

1. Find the speed of a car that travels 45 km in 30 minutes.

2. Find the distance in km travelled by an aeroplane moving at 80 m/s for 15minutes

3. How long will it take a cyclist to make 10 laps of a 400 m cycle track cycling at an average speed of 40 km/h?

I hope you got all of them correct, then continue and compare with the answers given below. If not, you should go through the ones you missed carefully and then compare with the given solutions.

Model Answers

Activity 13.3

1. You should write 30 minutes as $\frac{1}{2}$ hour

$\textit{The average speed} = 45km \div 30\,min$

Change 30 minutes into hours:

$$= 45km \div \frac{1}{2}h$$

Change from division to multiplication by inverting the second number:

$= 45km \times \frac{2}{h}$

$= 90\ km/h$

2. 15 minutes = $15 \times 60\ s = 900\ seconds$

$The\ distance\ travelled = 80\ m/s \times 900\ s$

$= 72\ 000\ m$

Converting 72 km to metres

$\boxed{1\text{ km} = 1000\text{ m}}$

$72\ km = 72 \times 1\ km = 72 \times 1000m = 72000m$

3. Total distance cycled = $10 \times 400\ m$

$= 4000\ m$

$Time\ taken = \frac{4000\ m}{40\ m/s}$

$= 100\ s$

You have now completed the subunit on Rate, do a quick review of the content of this subunit on Rate and then continue on to the next subunit on decrease and increase a quantity by a given ratio.

Decrease and increase a quantity by a given ratio

In this subunit we will look at how to decrease and increase a quantity by a given ratio. You have already been introduced to this concept in Unit 8, when learning about discounts in commercial mathematics. When a shop-owner decides to have a sale, prices are usually reduced by the same percentage for all goods in the shop. This idea is not restricted to shop-keepers but can be used in a variety of situations.

At the end of this subunit you should be able to:

- *increase and decrease* a quantity by a given ratio.

There are 8 pages in this subunit.

A quantity can be increased or decreased in a given ratio by being multiplied by a suitable fraction. For example,

if a price increases in the ratio 5: 3, then it is multiplied by the fraction $\frac{5}{3}$ and if a price is decreased in the ratio 4: 9, then it is multiplied by $\frac{4}{9}$.

When Christmas approaches, shopkeepers often charge the full price because demand among consumers is very high. After the Christmas period, shopkeepers want to get rid of any stock they didn't sell. The sale starts. Put items tend to increase their prices from the sale price to the full non-sale price.

Can you see any comparisons being made in this situation?

One can compare the full price before the sale and the price after the sale period.

Typically, prices drop during a sale as shopkeepers try to attract additional customers. This means that, compared to the normal price, the price during the sale is lower.

Figure 13.1 : Some of the common advertisements.

I'm sure you have come across advertisements (like those in figure 13.1) in shop windows and on pamphlets. Did you realise that there is a connection between these advertisements and the type of mathematics you are studying in this course?

The M500 above is reduced to M200.

The price is reduced in the ratio $\frac{200}{500} = \frac{2}{5}$ which we normally write as 2:5

M500.00 x $\frac{2}{5}$ = M200.00 (decreasing a quantity by a ratio)

Note that the seller does not put all of the information in the advertisement, so that it will not resemble school mathematics. As the buyer, you have to fill in the missing information to realise the mathematics involved. But if you know how to do the mathematics for yourself, you can make informed choices about whether to buy or not.

Now let us look at the following examples to emphasise the ideas of decreasing or increasing a quantity by a given ratio.

Example 4

Increase R450 in the ratio 7 : 3

Solution

$$R450 \times \frac{7}{3} = R1050$$

R450 is increased to R1 050, it is not increased by R 1050.

Example 5

Decrease 70 kg in the ratio 2: 5

Solution

$$70\ kg \times \frac{2}{5} = \frac{140}{5} kg$$

$$= 25 kg$$

70 kg is decreased to 25 kg; it is not decreased by 25kg.

Activity

Activity 13.4

1. Increase M2.10 in the ratio 9:7.

2. Increase 4.50 km in the ratio 7:5.

3. Decrease 750 ml in the ratio 10:25.

4. Decrease 96 apples in the ratio 5:6.

Model Answer

Activity 13.4

1. $M2.10 \times \frac{9}{7} = M2.70$

2. $4.50\ km \times \frac{7}{5} = 6.3\ km$

3. $750\ ml \times \frac{10}{25} = 300\ ml$

4. $96\ apples \times \frac{5}{6} = 80\ apples$

Note that the units in each question are the same. We use the idea of ratio only when sets or quantities of the same type are compared.

You have now completed the subunit on how to decrease and increase a quantity by a given ratio, do a quick review of the content of this subunit and then continue on to the next one.

Lesson 3 Proportion

When studying mathematics at junior secondary level, you learned about direct and inverse proportion.

Mr and Mrs Williams have 3 children. They have only one in school this year. The annual fees are M3 400. The second child is due for school next year. How much will they pay for the two children next year, assuming there will be no fees increase?

____

____

Mr and Mrs Williams will pay M 3 400 + M3 400 = M6 800.

The number of children in school next year will have increased. It seems even the money to be paid has increased.

The number of children has doubled, so has the fees. This is an example of direct proportion. In direct proportion, increasing one quantity increases another.

What happens in indirect or inverse proportion?

____

____

In indirect proportion, increasing one quantity decreases another.

At the end of this sub-unit you should be able to:

- *formulate* equations from direct and inverse proportion situations, and use the equations to solve for one quantity given others.

There are 13 pages on this subunit on variation and proportion.

Direct proportion

Proportion is a way of relating quantities.

Two items, seemingly unrelated, will be said to be directly proportional if increasing one increases another, or if decreasing one also decreases another.

Example 1

A bar of soap costs M7.95. How much will Tefo pay for 4 bars?

Solution

The more bars of soap are bought, the more money Tefo will have to pay.

Number of bars of soap	Cost
1	M7.95
2	M15.90
3	M23.85
4	M31.80

The money to be paid is said to be directly proportional to the bars of soap to be bought. This is normally written, Y α X, where in this case Y is the money to be paid, and X is the bars of soap.

We normally say

1 bar = M 7.95

2 bars = M15.90

3 bars = M23.85

4 bars = M31.80

To get the amounts on the right side of the equation, each of the quantities on the left has to be multiplied by M7.95

We can say:

1 bar x M7.95 = M 7.95

2 bars x M7.95 = M15.90

3 bars x M7.95 = M23.85

4 bars x M7.95 = M31.80

The M7.95 is a constant of proportionality. This is the constant of one bar.

This leads to the equation of direct proportion which is Y = kX, where k is the constant of proportionality.

Example 2

Bus fare for 15 children going on a school trip is M900. Mpho has two children on that trip. How much is she going to pay for them?

Solution

This is a case of direct proportion.

M900 = k15

$k = \frac{900}{15}$

k = 60

Each child is paying M60

Mpho will pay M60 × 2 children = M120

Example 3

Tebello sells milk for his uncle. The milk is sold in litres. Each litre of milk costs M5.00. Tebello is paid 8 lisente for each litre sold.

The table below shows the number of litres of milk that Tebello sold from Monday to Friday.

Number of litres of milk(x)	5	10	20	30	8
Tebello's income in lisente(y)	40	80	160	240	64

a) Find the ratio of Tebello's income to the number of litres sold.

b) If the number of litres of milk sold is 15, find Tebello's income.

c) If Tebello's income is 200 lisente, what is the number of litres of milk sold?

d) How is the value of y (Tebello's income) related to x (the number of litres of milk)?

Solution

a) 8 lisente: 1 litre

8:1

b) In direct proportion, the quantities increase or decrease by the same factor

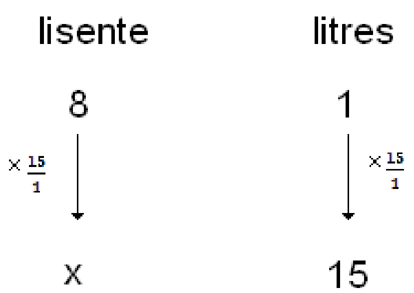
8 : 1

X: 15

1 has increased by the factor 15. 8 will have to change by the factor 15.

$8 \times 15 = 120$

Tebello's income is 120 lisente

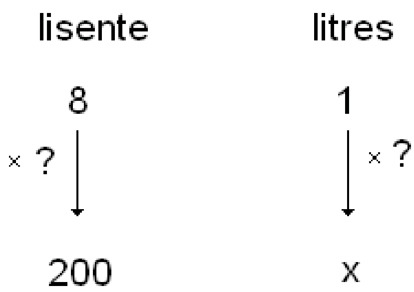


From the above ratio because 1x 15 = 15, then 8 x 15 = x.

Therefore, x = 120.

Tebello's income is 120 lisente

c)



From the information given you can write (to find the multiplying factor):

8 x ? = 200,

$? = \frac{200}{8}$

? = 25

Therefore, to get the number of litres sold you use the same multiplying factor found above. Thus,

1 x 25 = x

That is, x = 25

The number of litres of milk sold is 25.

d) $y = 8x$

If you were able to get at least three of them correct you can go through the next activity.

Note it

In the above activity we say the income is **directly proportional** to the number of litres of milk sold, or we can say the income **varies directly** as the number of litres of milk sold. Therefore, the ratio $\frac{y}{x} = 8$, which is a constant and represented by letter k.

The symbol $\propto$ is used to represent variation.

y **varies directly** as *x* is written as $y \propto x$.

$y \propto x$ means that $y = kx$, where *k* is constant and $k \neq 0$.

Therefore the above activity is an example of **direct proportion** or **direct variation.**

If the number of litres of milk sold increases, the income also increases.

If the number of litres of milk sold decreases, the income also decreases.

Inverse or Indirect Proportion

In indirect proportion, as one quantity increases the other decreases.

Example 1

10 men can do a certain piece of work in 4 days. Two fall sick. How long will it take the 8 men to complete the work, assuming they work at the same rate?

It goes without saying that the 8 men will take more than 4 days.

Fewer men will do the work in more than 4 days.

This is a case of indirect proportion.

Two quantities Y and X, are in inverse proportion if a factor k, that changes Y, changes X by the reciprocal of k, which is $\frac{1}{k}$.

We can say 4 days = 10 men

X days = 8 men

This is the question we ask ourselves. What changed 10 to 8?

The number of men is decreased by $\frac{8}{10}$

$10 \times \frac{8}{10}$

$= 8$ men

This is a case of indirect proportion. So the change to the number of days should be brought by the inverse of $\frac{8}{10}$ which is $\frac{10}{8}$.

So the number of days is increased in the ratio $\frac{10}{8}$

So time taken by 8 men $= 4 \times \frac{10}{8}$

$= 5$ days

Number of men	Days taken to do the work
10	4
8	5

The number of men, Y, is indirectly proportional to the number of days, X, taken to do the work. This is normally written, $Y \; \alpha \; \frac{1}{X}$.

This leads to the equation of indirect proportion which is $Y = k\frac{1}{X}$.

$$Y = \frac{k}{X}$$

Example 2

The given table shows the number of days that can be taken in a boarding school to feed pupils with the remaining food.

Number of pupils(x)	5	20	25	60
Number of days(y)	120	30	24	10

a) Multiply each pair of numbers in the table above. What do you realise?

b) Find the number of pupils that the school would feed in 40 days.

c) Find the number of days the school will take to feed 100 pupils.

d) How long would the food last to feed 50 pupils?

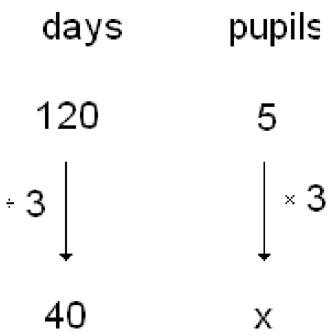
e) How many pupils would the school be able to feed for 85 days?

Solution

a) $5 \times 120 = 600, 20 \times 30 = 600, 25 \times 24 = 600, 60 \times 10 = 600$

When multiplying each pair the result is always 600.

b)



From the provided information you can write:

120 x ? = 40 (to find the ratio)

$? = \frac{40}{120}$ $(= \frac{1}{3})$ (this is the ratio).

Then, to find the number of pupils to be fed in 40 days we use the inverse of the ratio (i.e. divide by $\frac{40}{120}$ or $\frac{1}{3}$ or multiply by $\frac{120}{40}$ or 3). Therefore, you get

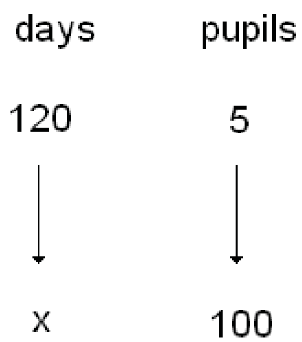
$5 \div \frac{1}{3} = x$ (or alternatively $5 \times 3 = x$)

$5 \div \frac{1}{3} = 15$ (or alternatively $5 \times 3 = 15$)

X = 15

Hence, the number of pupils that can be fed in 40 days is 15.

c)



From the information given you can write:

$5 \times ? = 100$ (to find the multiplying ratio)

$? = \frac{100}{5}$ (this is the ratio (also = 20))

Then, to find the number of days 100 pupils will be fed you use (multiply by) the inverse ratio or divide by the same ratio.

Since the food will last 5 pupils for 120 days, you must decrease the number of days in the ratio 5:100.

$$y = 120 \times \frac{5}{100}$$

$$= 6 \text{ days}$$

The school would be able to feed 100 pupils for 6 days.

d)

days	pupils
30	20
x	50

Since the food will last 20 pupils for 30 days, you must decrease the number of days in the ratio 20:50.

$$y = 30 \times \frac{20}{50}$$

$$= 12 \text{ days}$$

The school would be able to feed 50 pupils for 12days.

e)

days	pupils
30	20
↓	↓
85	x

Since the food will last 20 pupils for 30 days, you must decrease the number of pupils in the ratio 30:85.

X = 20 x $\frac{30}{85}$

= 7.05 ≈ 7 (no pupils fractions)

Therefore, the school will be able to feed 7 pupils for 85 days.

Note that yx = 600, the number 600 is a constant.

We can also say y = $\frac{600}{x}$

Or say ,

$$y = 600\ \frac{1}{x}$$

This is **inverse variation** or **indirect proportion**. In this example the number of pupils is **inversely proportional** to the number of days.

In conclusion, note that we say *y* is **inversely proportional** to *x*. This can be written as $y \propto \frac{1}{x}$.

Therefore, $y \propto \frac{1}{x}$ **means** $y = \frac{k}{x}$ or $yx = k$, the number *k* is a constant.

Where $k \neq 0$

Activity

Activity 13.11

Answer all the questions.

1. Write an equation for each of the following statements:

a) For a given distance, the time, t, taken on a journey is inversely proportional to *v*, the speed.

b) The weight of an object *w*, varies inversely as the square of *r*, its distance from the centre of the earth.

2. (a) y is inversely proportional to x, and $y = 6$ when $x = 3$. Find the value of y when $x = 4$.

b) y varies as $\frac{1}{x^2}$, and when $x = 4, y = \frac{3}{4}$. Find the value of y when $x = 1$.

3. A car travelling at 80 km/h takes 6 hour to complete a journey.

a) How long would it take a car travelling at 120 km/h?

b) At what speed would a car finish a journey in 12 hours?

4. A quantity of water will last 20 chickens for 4 days. How long would it last for

a) 40 chickens? b) 80 chickens?

Model Answers

Activity 13.11

1.a) $t \propto \frac{1}{u}$, the equation is $t = k\frac{1}{u}$ or $t = \frac{k}{u}$

b) $w \propto \frac{1}{s^2}$, the equation is $w = \frac{k}{s^2}$

2.a) $y \propto \frac{1}{x}$, the equation is $y = \frac{k}{x}$ or $yx = k$

Substituting the values of x and y

$6 \times 3 = k$

$18 = k$ or $k = 18$

Now $y = \frac{18}{x}$

Substituting the value of x

$y = \frac{18}{4}$

$= 4\frac{1}{2}$

b) $y \propto \frac{1}{x^2}$, the equation is $y = \frac{k}{x^2}$ or $yx^2 = k$

Substituting the values of x and y

$\frac{3}{4} \times 4^2 = k$

$k = 12$

Now $y = \frac{12}{x^2}$

Substituting the value of x

$y = \frac{12}{1}$

$y = 12$

3.a) $k = 80\frac{km}{h} \times 6 \text{ hours}$

$= 480 \text{ km}$

$s \propto \frac{1}{t}$ therefore, $s = \frac{k}{t}$

Substituting the values of k and s

$120 = \frac{480}{t}$

$t = \frac{480}{120}$

$= 4 \text{ hours}$

The journey would take 4 hours.

b) $s \propto \frac{1}{t}$ therefore, $s = \frac{k}{t}$

Substituting the values of k and t

$s = \frac{480}{12}$

$= 40 km/h$

A car would take 40 km/h to finish the journey.

4.a) $k = 20 \times 4 = 80$

Number of chickens (c) varies indirectly as number of days (d).

$c \propto \frac{1}{d} \text{ or } c = \frac{k}{d}$

Substituting the values of k and c

$40 = \frac{80}{d}$

$d = 2$

The water would last for 2 days.

b) $k = 20 \times 4 = 80$

Number of chickens varies indirectly as number of days.

$c \propto \frac{1}{d}$ or $c = \frac{k}{d}$

Substituting the values of k and c

$80 = \frac{80}{d}$

$d = 1$

The water would last for 1 day.

You have now completed work on this unit on Ratio, Rate, and Proportion. Do a quick review of the entire content of this unit and then continue on to the unit summary.

Unit Summary

Summary

In this unit you learned that:

1. A ratio is a fraction that expresses a relationship between two like quantities.
2. The ratio *m* to *n* can be written in the form $m : n$ or $\frac{m}{n}$
3. A ratio between two quantities is usually expressed in its simplest form, like a fraction.
4. Ratios can also be used to compare more than two like quantities. In these cases, the ratio is normally written in the form $m : n : o : p$.
5. When a quantity is increased in a given ratio, it becomes more than the original quantity.
6. When a quantity is decreased by a given ratio, it becomes less than the original quantity.
7. The symbol $\propto$ is used to represent proportion.
8. *y* **varies directly** as *x* is written as $y \propto x$

$y \propto x$ means that $y = kx$, where *k* is constant and $k \neq 0$

9. *y* is **inversely proportional** to *x*. This can be written as $y \propto \frac{1}{x}$

Therefore, $y \propto \frac{1}{x}$ **means** $y = \frac{k}{x}$ or $yx = k$, the number *k* is a constant.

Where $k \neq 0$.

You have completed the material for this unit on Ratio, Proportion, and Rate. You should spend some time reviewing the content in detail.

Once you are confident that you can successfully write an examination on the concepts, try the assignment. Check your answers with those provided and clarify any misunderstandings.

Your last step is to complete the assessment. Once you have completed the assessment, proceed to the next unit, which looks at different properties of the Circle.

Assignment

Assignment

1. **Answer all questions.**
2. **Show all of the work you do to arrive at an answer.**

Total marks = 30 **Time**: 60 mins

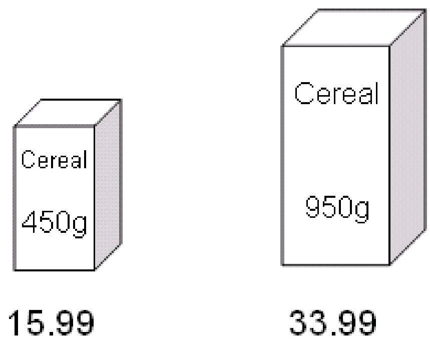
.

The marks allotted to each question or part of a question is shown in (parentheses).

1. (a) Write $\frac{30}{72}$ in its simplest form. (2)

   (b) Write 25:40:15 in its simplest form. (2)

2. Breakfast cereal is available in two different sizes from our shops.



(a) Work out the cost of 100g of Cereal when bought in the 450g box (to the nearest cent). (2)

(b) Work out the cost of 450g when bought in the 950g box (to the nearest cent). (2)

(c) Which is the better buy? Give one reason in support of each size. (2)

3. Three men can build a wall in 10 hours. How many men will be needed to build the wall in 6 hours? (3)

4. Two boys Thabo and Lebo share the cost of a meal in the ratio 3:4. Lebo pays M37.40. What is the total cost of the meal? (3)

5. Given that the quantity y is inversely proportional to another quantity x, and that $y = 10$ when $x = 20$; find y when $x = 0.01$
(4)

6. Two brothers Khoali and Khaka bought shares costing M140 000.00 in the stock market. Khoali paid M60 000.00 and Khaka paid M80 000.00. They decided to sell the shares for M168 000.00. How much of M168 000.00 will Khaka get? (4)

7. The cost of making a telephone call is proportional to the time of call. If the telephone company charges 63 lisente for a 7 minutes call, how much will a 3 minutes call be charged? (6)

Model Answers to Assignment

Question 1

(a) $\frac{30}{72} = \frac{5}{12}$

(b) 25 : 40: 15

5 : 8: 3

Question 2

$\frac{100 \times 15.99}{450} = 3.55$

100g costs 3.55

$\frac{450g \times 33.99}{950g} =$ 16.10

450g costs 16.10

Buying 450g for 15.99 is a better deal

Question 3

$\frac{3 \times 10}{6}$ = 5 men

Question 4

Thabo pays 3 parts and Lebo pays 4 parts of the peal.
4 parts of the meal cost M37.40.
So, 1 part costs M37.40 ÷ 4 = M9.35
Therefore, total cost of the meal (3+4 =7) = M9.37 × 7 = M65.45

Question 5

y= $\frac{k}{x}$

$$10 = \frac{k}{20}$$

k = 200

When x = 0.01; $y = \frac{200}{0.01}$

y= 20 000

Question 6

Ratio of shares	Khoali : Khaka M60 000.00 : M80 000.00 3 : 4
Ratio of profit	Khoali : Khaka 3:4
Total profit	3 + 4 = 7
Khaka share of profit	$\frac{4}{7} x M168\ 000.00 = M96\ 000.00$

Question 7

Let Cost of call be C

Let Time of call be t

So, $C \propto t$

That is, $C = Kt$

To find the value of K

C = Kt

63 = K (7)

$\frac{63}{7} = K$

$9 = K$

This means C = 9t

Charge of 3 minutes call

$C = 9(3)$

$C = 27$

3 minutes call is charged 27 lisente

Assessment

Assessment

1. **Answer all questions.**
2. **Show all of the work you do to arrive at an answer.**

Total marks = 24 **Time**: 80 mins

The marks allotted to each question or part of a question are shown in (parentheses).

1. A bag contains a number of balls, some are blue and some are black. Three quarters of the balls are blue. What is the ratio of the blue to black balls? (2)

2. (a) Write each ratio in its simplest form

(i) $\frac{4}{9} : \frac{5}{4} : \frac{3}{5}$

(ii) 3 : 0.24 : 0.6

(4)

(b) Find the value of x and y in the following ratios:

(i) 6 : 8 = 15: x

(ii) $\frac{2}{3} : 1 = y:4$

(4)

(c) If 12 men can plough a field in 8 days, how many men can do the job in 2 days if they work at the same rate?

(2)

3. (a) 100 g of spaghetti contains 3.6 g of fibre.
Express mass of fibre : mass of spaghetti as a ratio of two integers in its simplest form.
(2)

4. Lineo bought 5 maize cobs for M5.00 each. At the same price, how would 10 maize cobs costs? (2)

5. Calculate
   a. A packet of apples is sold for M12.99 and there were 9 apples in the packet. What is the cost per apple? (2)

   b. If 1.3 Kg of mince meat costs M42.84. What is the price per Kilogram? (2)

c. If you used 22 Kilolitre (Kl) of water in June and the cost was M144.98. What is the price of water per kilolitre?
(2)

d. If petrol costs M8.24 per litre, how much would it cost to fill a 50 litre tank?

(2)

Answers

1. Three quarters of balls are blue. So one quarter of balls are black

$\frac{3}{4} : \frac{1}{4}$

Multiply by 4 to simplify the ratio to get

$3 : 1$

2.

(a)

(i) $180 \times \frac{4}{9} \quad : \quad \frac{5}{4} \times 180 \quad : \quad \frac{3}{5} \times 180$

80: 225 : 108

(ii)

$3 \times 100: 0.24 \times 100: 0.6 \times 100$

300: 24: 60
25 : 2 : 5

(b)

(i) 6 : 8 = 15: x

$$\frac{6}{8} = \frac{15}{x}$$

$$x = \frac{15 \times 8}{6}$$
$$= 20$$

(ii) $\frac{2}{3} : 1 = y:4$

$$\frac{\frac{2}{3}}{1} = \frac{y}{4}$$

$$y = \frac{\frac{2}{3} \times 4}{1}$$

$$y = \frac{8}{3}$$

(c) If 12 men can plough a field in 8 days, how many men can do the job in 2 days if they work at the same rate?

$$x = \frac{12 \times 8}{2}$$
$$= 48 \text{ men}$$

3.

3.6 g : 100g
3.6 : 100
1 : 27.78
1 : 27

4. 5 maize cobs – M5.00
10 maize cobs - ?

$? = \frac{10}{5}$ x M5.00
= M10.00

5. (a) 9 apples – M12.99
1 apple - ?

$? = \frac{1}{9}$ x M12.99
= M 1.44

(b) 1.3 Kg of mince meat – M42.84
1 Kg of mince meat - ?

$? = \frac{1}{1.3} x M42.84$
$= M32.95$

(c) 22 Kilolitre of water – M144.98
1 Kilolitre - ?

$? = \frac{1}{22} x\ M144.98$
$= M6.59$

(d) 1 litre – M8.24

50 litre - ?

? = 50 x M8.24

= M412.00