Unit 2 Paper 3 (2021)

1(a)

x	e^{-x^2}	
-1	0.3678	
-0.75		1.7551
-0.50		1.2840
-0.25		1.0645
-0	1	
	1.3678	4.1036

$$\int_{-1}^{0} e^{-x^2}\,dx = \frac{1}{2}(0.25)\left[1.3678 + 2(4.1036)\right]$$

$$= 1.1969$$

(b) $z = 8-6i$

let $q = x+iy = \sqrt{8-6i}$

$(x+iy)^2 = 8-6i$

$x^2 - y^2 + 2xyi = 8-6i$

$x^2 - y^2 = 8$

$2xy = -6$

$x = -\frac{3}{y}$ $\quad x^2 = \frac{9}{y^2}$

$\frac{9}{y^2} - y^2 = 8$

$9 - y^4 = 8y^2$

$$y^4 + 8y^2 - 9 = 0$$

$$(y^2 - 1)(y^2 + 9) = 0$$

$y = \pm 1$ $\quad$ $y = \sqrt{-9}$

not a solution

$y = \pm 1 \Rightarrow x = \mp 3$

$q = -3 + i$ or $q = 3 - i$

(c) $$V = \frac{27xy - x^2y^2}{2(x+y)}$$

$$\frac{\partial V}{\partial x} = \frac{\cancel{2}(x+y)(27y - 2xy^2) - (27xy - x^2y^2)\cancel{2}}{4(x+y)^2}$$

$$= \frac{(x+y)(27y - 2xy^2) - (27xy - x^2y^2)}{2(x+y)^2}$$

$$= \frac{\cancel{27xy} + 27y^2 - 2x^2y^2 - 2xy^3 - \cancel{27xy} + x^2y^2}{2(x+y)^2}$$

$$= \frac{27y^2 - x^2y^2 - 2xy^3}{2(x+y)^2} = 0$$

$$y^2(27 - x^2 - 2xy) = 0$$

1(c) $V = \frac{27xy - x^2y^2}{2(x+y)}$

$$\frac{\partial V}{\partial y} = \frac{\cancel{2}(x+y)(27x - 2x^2y) - (27xy - x^2y^2)\cancel{2}}{4(x+y)^2}$$

$$= \frac{27x^2 + 27xy - 2x^3y - 2x^2y^2 - 27xy + x^2y^2}{2(x+y)^2}$$

$$= \frac{27x^2 - 2x^3y - x^2y^2}{2(x+y)^2}$$

$$= \frac{x^2(27 - 2xy - y^2)}{2(x+y)^2} = 0$$

$$27 - x^2 - 2xy = 0$$
$$27 - 2xy - y^2 = 0$$

$$-x^2 + y^2 = 0$$
$$x^2 = y^2 \Rightarrow x = y$$

$$27 - x^2 - 2xx$$
$$27 - 3x^2 = 0$$
$$3x^2 = 27$$
$$x^2 = 9$$
$$x = 3 \quad y = 3$$

$$z = \frac{27 - (3)(3)}{2(3+3)} = \frac{27-9}{12} = \frac{18}{12} = \frac{3}{2}$$

2 (a) $0.\overline{65} = 0.65 + 0.0065 + 0.000065 \ldots$

$$= \frac{65}{100} + \frac{65}{100}\left(\frac{1}{100}\right) + \frac{65}{100} \times \left(\frac{1}{100}\right)^2$$

$$= + \ldots \frac{65}{100}\left(\frac{1}{100}\right)^{n-1}$$

$$a = \frac{65}{100} \qquad r = \frac{1}{100}$$

$$0.\overline{65} = S_\infty = \frac{\frac{65}{100}}{1 - \frac{1}{100}} \quad \frac{\frac{65}{100}}{\frac{99}{100}} = \frac{65}{99}$$

(b)(i) $\left(1 + \frac{x}{3}\right)^5 = 1 + 5\left(\frac{x}{3}\right) + \frac{(5)(4)\left(\frac{x}{3}\right)^2}{2!}$

$$= 1 + \frac{5x}{3} + \frac{20}{18}x^2$$

$$= 1 + \frac{5x}{3} + \frac{10}{9}x^2$$

(ii) $1.033^5 = 1 + \frac{33}{1000}$

$$= 1 + \frac{1}{3}\left(\frac{99}{1000}\right)$$

$$x = 0.099$$

$$1.033^5 = 1 + \frac{5}{3}(0.099) + \frac{10}{9}(0.099)^2$$

$$= 1.176$$

2 (c) $f(x) = 2x\sin x - 3$

$f'(x) = 2x\cos x + 2\sin x$

$$x_{n+1} = x_n - \frac{2x\sin x - 3}{2x\cos x + 2\sin x}$$

$x_1 = 1$

$$x_2 = 1 - \left(\frac{2\sin 1 - 3}{2\cos 1 + 2\sin 1}\right) = 1.4766$$

$$x_3 = 1.4766 - \frac{2(1.4766)\sin(1.4766) - 3}{2(1.4766\cos(1.4766) + 2\sin(1.4766)}$$

$= 1.5034$

$$x_4 = 1.5034 - \frac{2(1.5034)\sin(1.5034) - 3}{2(1.5034)\cos(1.5034) + 2\sin(1.5034)}$$

$= 1.50$

3 (i)

$$x + y + z = 300$$
$$30x + 40y + 50z = 11000$$
$$10x + 15y + 40z = 6000$$

(ii)

$$\begin{pmatrix} 1 & 1 & 1 \\ 30 & 40 & 50 \\ 10 & 15 & 40 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 300 \\ 11000 \\ 6000 \end{pmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 300 \\ 30 & 40 & 50 & 11000 \\ 10 & 15 & 40 & 6000 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 300 \\ 3 & 4 & 5 & 1100 \\ 2 & 3 & 8 & 1200 \end{array}\right] \begin{array}{l} \\ R_2/10 \\ R_3/5 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 300 \\ 0 & 1 & 2 & 200 \\ 0 & 1 & 6 & 600 \end{array}\right] \begin{array}{l} \\ R_2 = R_2 - 3R_1 \\ R_3 = R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 300 \\ 0 & 1 & 2 & 200 \\ 0 & 0 & 4 & 400 \end{array}\right] \quad R_3 = R_3 - R_2$$

$4z = 400 \Rightarrow z = 100$

$y + 2z = 200$

$y + 200 = 200 \Rightarrow y = 0$

$x = 200$

(b) no of possible committee

$= {}^{10}C_6 = 210$

no of committees with NO men

$= {}^{7}C_6 = 7$

no of committee with AT LEAST one man

$= {}^{10}C_6 - {}^{7}C_6 = 210 - 7 = 203$

(ii) no. of committee with 5 women

$= {}^7C_5 \cdot {}^3C_1$

$= 21 \cdot 3 = 63$

Probability $= \dfrac{63}{203}$

$= 0.31034$