Preview Unit 2 Test 3

1. A bag contains 9 discs numbered $1, 2, 3, 4, 5, 6, 7, 8, 9$.
   i. Andrea chooses 5 discs at random, without replacement, and places them in a row.
      a) How many different 5-digit numbers can be made? [2]
      b) How many different ODD 5-digit numbers can be made? [2]

   ii. Andrea's 5 discs are put back in the bag. Martin chooses 5 discs at random, without replacement. Give your answers as **EXACT** values, find the probability that
      a) The 5 digits include at least 4 odd digits [6]
      b) The 5 digits add up to 33. [3]

2. a) Find the general solution of the differential equation

$$x\frac{dy}{dx} + 2y = 4x^2$$

   [7]

   b) Hence, find the particular solution for which $y = 5$ at $x = 1$, giving your answer in the form $y = f(x)$. [3]

3. Given the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = e^{-x}$$

   Find

   (a) its complementary function [4]

   (b) its particular integral [6]

   (c) its general solution [1]

   Given that $y = 0$ and $\frac{dy}{dx} = 0$ when $x = 0$.

(d) Find the particular solution for the differential equation [6]

4. $A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 5 & 3 & u \end{pmatrix}$

(a) Show that $|A| = 2(u - 1)$ [4]

(b) Hence, find the value of u if A is a singular matrix. [2]

Given that A is a non-singular matrix

(c) find A^{-1}, in terms of u, using the cofactors method. [7]

5. $x - 2y + 3z = 4$
$2x + y - 4z = 3$
$-3x + 4y - z = -2$

For the system of equations above

(a) Write the augmented matrix. [1]

(b) Reduce the augmented matrix obtained to echelon form. [3]

(c) Solve for *x*, *y* and *z*. [3]

Answers

Question 1

i.
(a) 15120
(b) 8400

ii.
(a) $\frac{1}{6}$

(b) $\frac{1}{63}$

Question 2

(a) $y = x^2 + \frac{c}{x^2}$

(b) $y = x^2 + \frac{4}{x^2}$

Question 3

(a) $y = Ae^{-4x} + Be^{2x}$

(b) $y = -\frac{1}{5}e^{-x}$

(c) $y = Ae^{-4x} + Be^{2x} - \frac{1}{5}e^{-x}$

(d) $y = \frac{1}{10}e^{-4x} + \frac{1}{10}e^{2x} - \frac{1}{5}e^{-x}$

Question 4

(b) $u = 1$

(c) $\frac{1}{2(u-1)}\begin{pmatrix} u-3 & -u-3 & 2 \\ 5-u & 3\,u+5 & -4 \\ -2 & -4 & 2 \end{pmatrix}$

Question 5

(a) $\left(\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 2 & 1 & -4 & 3 \\ -3 & 4 & -1 & -2 \end{array}\right)$

(b) $\left(\begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & 5 & -10 & -5 \\ 0 & 0 & 20 & 40 \end{array}\right)$

(c) $x = 4 \;\; y = 3 \;\; z = 2$