Preview Unit 2 Test 2 (2017)

1)) Write down the n^{th} term of the sequence $3, 7, 11, 15, \ldots$ [1]

b) Hence, show that the sum of the first *n* terms of the series $3^2 + 7^2 + 11^2 + 15^2 + \cdots$ is given by

$\frac{1}{3}n(16n^2 + 12n - 1)$ [8]

2) The first three terms of a geometric sequence are $\cos x$, $\sin 2x$ and $4\cos x \sin^2 x$,

a) Find the common ratio r, in its simplest form. [3]

Given that $x = \sin^{-1}\left(\frac{1}{4}\right)$, $x > 0$

b) Show that the sum to infinity of the series is $\frac{\sqrt{15}}{2}$. [4]

3) a) Given that

$$f(r) = r!$$

show that

$$f(r+2) - f(r+1) = (r+1)^2 \times r!$$

[3]

b) Hence find

$$\sum_{r=1}^{n} [(r+1)^2 \times r!]$$

[5]

4) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 5y = 0$

a) Find an expression for $\frac{d^3y}{dx^3}$ in terms of $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$ and y. [5]

Given that $y = 2$ and $\frac{dy}{dx} = 2$ at $x = 0$.

b) Find the power series for *y*, in ascending powers of *x*, up to and including the term in x^3. [6]

5) $f(x) = (1+5x)^{-1}$, $|x| < \frac{1}{5}$.

a) Expand $f(x)$ in ascending powers of *x* up to and including the term in x^3. [5]

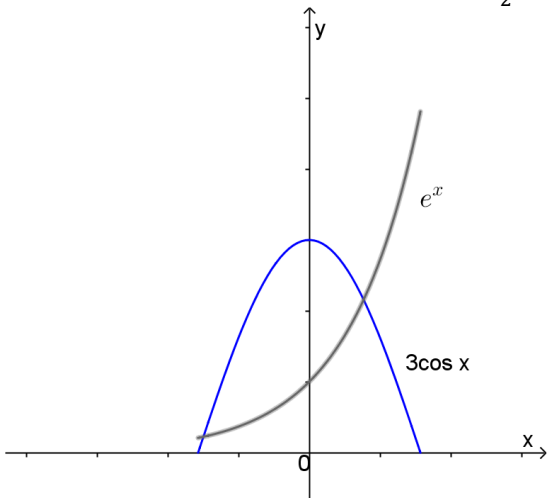
b) Hence show that,

$$\frac{1+x}{1+5x} \approx 1 - 4x + 20x^2 - 100x^3$$

[3]

c) By taking a suitable value for *x*, which should be stated, use the series expansion in part b) to find an approximate value for $\frac{101}{105}$, giving your answer to 5 decimal places. [5]

6) The figure below shows the graph of $y = 2\cos x$ and $y = e^x$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



Given that $f(x) = e^x - 3\cos x$

a) Write down the number of solutions of the equation $f(x) = 0$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [1]

b) Show that the equation $f(x) = 0$ has a solution, α, in the interval $[0, 1]$. [4]

c) Using 0.7 as a first approximation to α, use the Newton-Raphson process once to find an improved estimate for α, giving your answer correst to 2 decimal places. [4]

There is another root, β, of the equation $f(x) = 0$ in the interval $[-2, -1]$.

d) Use linear interpolation once on this interval to estimate the value of β, giving your answer correct to 2 decimal places. [3]

Answers

Question 1

a) $4n - 1$

b) $\frac{1}{3}n(4n^2 + 15n + 8)$

Question 2

a) $2\sin x$

b)

Question 3

b) $(n + 2)! - 2$

Question 4

a) $\frac{d^3y}{dx^3} = \frac{-3\frac{d^2y}{dx^2}\left(\frac{dy}{dx}\right) - 5\frac{dy}{dx}}{y}$

b) $y = 2 + 2x - \frac{7}{2}x^2 + \frac{8}{3}x^3$

Question 5

a) $1 - 5x + 25x^2 - 125x^3$

c) 0.96190

Question 6

a) 2 solutions

c) 0.77

d) -1.48