Preview Unit 2 Test 2

1. A geometric progression has first term $\log_3 16$ and common ratio $\log_3 x$.
   (a) Find the set of values of *x* for which the geometric progression has a sum to infinity. [2]
   (b) Find the **EXACT** value of *x* for which the sum to infinity of the geometric progression is 4. [7]

2. (a) Write down the first four terms of Maclaurin expansion for $\ln(1+x)$. [1]
   (b) Hence, determine the first four terms for the expansion of $\ln(1+x^3)$. [2]
   (c) By using your result from (i) find the **EXACT** value of

$$1-\frac{1}{2}\left(\frac{1}{3}\right)^3+\frac{1}{3}\left(\frac{1}{3}\right)^6-\frac{1}{4}\left(\frac{1}{3}\right)^9+\cdots$$

[5]

3. (a) Express $\frac{2}{(r+2)(r+4)}$ in partial fractions. [5]
   (b) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n}\frac{2}{(r+2)(r+4)}=\frac{n(an+b)}{12(n+3)(n+4)}$$

   where *a*, and *b* are constants to be found. [8]

   (c) Hence show that

$$\sum_{r=n+1}^{2n}\frac{2}{(r+2)(r+4)}=\frac{24n^3+126n^2+150n}{12(n+2)(n+3)(n+4)(2n+3)}$$

[5]

4. (a) Find the binomial expansion of $\sqrt{1-4x}$, in ascending powers of *x* up to and including the term in x^3, simplifying each term. [4]
   (b) State the values of *x* for which the expansion is valid. [1]
   (c) Substitute $x=\frac{1}{100}$ into the binomial expansion in part (a) and obtain an approximation to $\sqrt{6}$. Give your answer correct to **5** decimal places. [5]

5. (a) Show that $\frac{d}{dx}(3^x) = 3^x \ln 3$ [4]

(b) Given that $f(x) = 3^x - x - 6$

i. Show that the equation $f(x) = 0$ has a root α in the interval $[1,2]$. [4]

ii. Use linear interpolation in the interval $[1,2]$ to find an approximation to α. Give your answer as an **EXACT** value. [3]

iii. Taking $x_1 = 1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to **3** decimal places. [4]

Answers

Question 1

(a) $\frac{1}{3} < x < 3$

(b) $x = \frac{3}{2}$

Question 2

(a) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$

(b) $x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \frac{x^{12}}{4} + \cdots$

(c) $27 \ln \frac{28}{27}$

Question 3

(a) $\frac{1}{r+2} - \frac{1}{r+4}$

(b) $\frac{n(7n+25)}{12(n+3)(n+4)}$

Question 4

(a) $1 - 2x - 2x^2 - 4x^3$

(b) $-\frac{1}{4} < x < \frac{1}{4}$

(c) 2.44949

Question 5

ii. $\frac{9}{5}$

iii. 1.887