HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2015
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 2 TEST 2
1 hour 20 minutes

This examination paper consists of **2** printed pages.
This paper consists of **8** questions.
The maximum mark for this examination is **53**.

INSTRUCTIONS TO CANDIDATES

(i) Write **in ink**
(ii) Write your name clearly on each sheet of paper used
(iii) Answer **ALL** questions
(iv) Number your questions identically as they appear on the question paper and do **NOT write your solutions to different questions** beside each other
(v) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

1) A sequence of positive integers $u_1, u_2, u_3, \ldots$ is given by $u_1 = 2$ and $u_{n+1} = 2u_n$ for $n \geq 1$.

(i) Write down the first four terms of this sequence. [1]

(ii) State what type of sequence this is, and express u_n in terms of n. [2]

2) Dominique has been a marathon runner for many years. She ran her first marathon in approximately 5 hours. She trained intensively and each marathon she was able to decrease her time by 2.5%.

(i) Approximately how many hours should it take Dominique to complete her 8th marathon? [2]

(ii) How many hours in total would she have run after she completed her 8th marathon? [2]

(iii) Her ultimate goal is to run a marathon in 4 hours. If she maintains her training schedule, how long should it take her to accomplish her goal? (Round off to the nearest marathon.) [2]

3) Use Maclaurin's Theorem to expand the function $e^{\cos x}$, in ascending powers of x as far as the term in x^2. [8]

4) Using the Taylor series, expand y up to terms in $(x-1)^3$, where $\frac{d^2y}{dx^2} + y\frac{dy}{dx} = x$ given that $y = 0$ and $\frac{dy}{dx} = 1$ at $x = 1$. [7]

5) Without expanding $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ completely, find

(i) the term independent of x [4]
(ii) the coefficient of x^6. [3]

6) Given $f(x) = e^x - 2x^2$

(i) Show that the equation $f(x) = 0$ has a root α in the interval $[-1, 0]$. [2]
(ii) By taking an initial approximation to α to be -0.5, use the Newton-Raphson method to find a second approximation to α, giving your answer correct to 3 significant figures. [4]

7) (i) Express $\frac{1}{(r+3)(r+1)}$ in partial fractions. [4]

(ii) Hence prove, by the method of differences, that

$$\sum_{r=1}^{n} \frac{2}{(r+3)(r+1)} = \frac{n(5n+13)}{6(n+2)(n+3)}$$ [5]

8) Prove by the method of mathematical induction that $(1+x)^n \geq 1 + nx$ for $n \geq 0$. [7]

<u>ANSWERS</u>

1) (i) 2, 4, 8, 16 (ii) geometric; $u_n = 2^n$
2) (i) 4.2 hrs (ii) 36.7 hrs (iii) approx 10 marathons
3) $e - \frac{e^2}{2}x^2$
4) $f(x) = (x-1) + \frac{(x-1)^2}{2!}$
5) (i) $\frac{7}{18}$ (ii) $\frac{189}{16}$
6) -0.541
7) $\frac{1}{2(r+1)} - \frac{1}{2(r+3)}$