

SBA preview Unit 2 Test 1 (2018)

1. The complex number $3+2i$ is denoted by z .

Find

- i. $|z|$
- ii. $\arg z$
- iii. $\frac{z}{4-i}$

2. The loci C_1 and C_2 are given by $\arg(z + 2i) = \frac{1}{4}\pi$ and $|z| = |z - 8|$ respectively

- i. Sketch on a single Argand diagram the loci C_1 and C_2 .
- ii. Indicate, by shading, the region of the Argand diagram for which $0 \leq \arg(z + 2i) \leq \frac{1}{4}\pi$ and $|z| \leq |z - 8|$

3. Given that

$$x = \csc^2 2y, \quad 0 < y < \frac{\pi}{6}$$

- a) Find $\frac{dx}{dy}$ in terms of y .
- b) Hence show that

$$\frac{dy}{dx} = -\frac{1}{4x(x-1)^{\frac{1}{2}}}$$

4. Given that

$$f(x) = \frac{3x^2 + 16}{(2+x)^2(1-3x)}, \quad |x| < \frac{1}{3}$$

- a) Express $f(x)$ as a sum of partial fractions [6]
- b) Hence find $\int f(x) dx$ [

5.

$$f(x) = (x^2 + 1)\ln x, \quad \text{where } x > 0$$

- i. Use differentiation to find the value of $f'(x)$ at $x = e$, leaving your answer in terms of e .
- ii. Find the exact value of $\int_1^e f(x) dx$

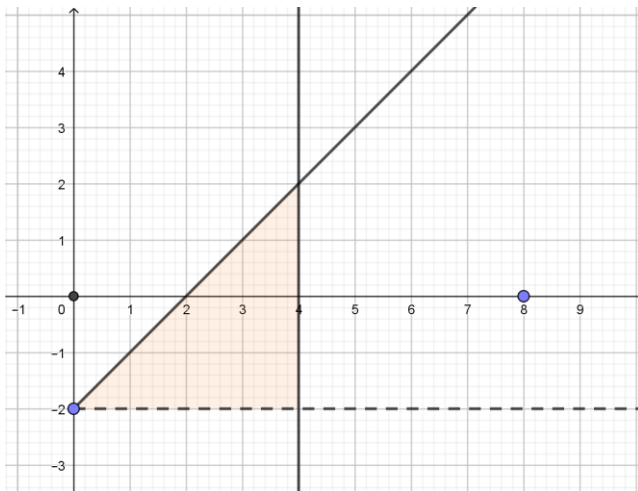
6. Use the substitution $u = 3^x$ to find the exact value of $\int_0^1 \frac{3^x}{(3^x+1)^2} dx$

Answers

Question 1

- i. $\sqrt{13}$
- ii. 0.588 rad
- iii. $\frac{4+7i}{5}$

Question 2



Question 3

a) $-4 \csc^2 2y \cot 2y$

Question 4

- a) $\frac{3}{1-3x} + \frac{4}{(2+x)^2}$
- b) $-\ln|1-3x| - \frac{4}{2+x} + c$

Question 5

- i. $3e + \frac{1}{e}$
- ii. $\frac{2}{9}e^3 + \frac{10}{9}$

Question 6

$$\frac{1}{2 \ln 3} - \frac{1}{4 \ln 3}$$