HARRISON COLLEGE INTERNAL EXAMINATION 2016
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
PREVIEW
PURE MATHEMATICS
UNIT 2 – TEST 1
1 hour 20 minutes

This examination paper consists of 2 pages.
This paper consists of 3 questions.
The maximum marks for this examination is 60.

INSTRUCTIONS TO CANDIDATES

1. Write your name clearly on each sheet of paper used.
2. Answer **ALL** questions.
3. Do **NOT** do questions beside one another.
4. Unless otherwise stated in the question, any numerical answer that is not exact **MUST** be written correct to **three** (3) significant figures.

EXAMINATION MATERIALS ALLOWED

1. Mathematical formulae sheet
2. Scientific Non-programmable calculator (non-graphical)

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1. (a) (i) Show that $\frac{dy}{dx} = -1$ at the point $A\left(1, \frac{1}{2}\right)$ on the curve

$$x + 2y - tan^{-1}(2y) = 2 - \frac{1}{4}\pi,$$ [3]

(ii) Find the value of $\frac{d^2y}{dx^2}$ at A. [Ans: -2] [4]

(b) The curve C is defined parametrically by

$$x = t^2 - 2\ln t\ , \quad y = 4(t-1) \quad t \in \mathbb{R} \quad t \geq 1$$

Find $\frac{d^2y}{dx^2}$ in terms of *t*. [Ans: $\frac{-t(t^2+1)}{(t^2-1)^3}$] [5]

(c) Given that $z = e^{x^2 - xy}$, show that

$$\frac{1}{z}\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial^2 z}{\partial y\, \partial x} + z$$ [5]

Total: 17 marks

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2. (a) Find (i) $\int x^2 \cos 3x \ dx$ [Ans : $\frac{x^2}{3}\sin 3x + \frac{2}{9}x\cos 3x - \frac{2}{27}\sin 3x + constant$] [5]

(ii) $\int \frac{x}{\sqrt{1-49x^4}}\ dx$ [Ans: $\frac{1}{14}sin^{-1}(7x^2) + constant$] [4]

(b) Let $f(x) = \frac{3x^2+5x+4}{x(x+1)^2}$

i. Express $f(x)$ in partial fractions. [Ans: $f(x) = \frac{4}{x} - \frac{1}{(x+1)} - \frac{2}{(x+1)^2}$] [4]

ii. Hence show that $\int_1^2 f(x)\ dx = \ln\frac{32}{3} - \frac{1}{3}$. [5]

(c) Use the trapezium rule with 4 strips to find an approximation to

$$\int_0^{\frac{2\pi}{3}} \sqrt{\cos\left(\frac{1}{2}x\right)}\ dx$$

giving your answer to 2 decimal places. [Ans: 1.89] [4]

Total: 22 marks

3. (a) Given $|z| = 2\sqrt{5}$ find the complex number *z* that satisfies the equation

$\frac{25}{z} - \frac{15}{z^*} = 1 - 8i$ [Ans: z = $2 + 4i$] [5]

(b) (i) Solve the equation $z^3 = 1 + \sqrt{3}i$ [Ans: $z = \sqrt[3]{2}e^{\frac{\pi}{9}}, \sqrt[3]{2}e^{\frac{7\pi}{9}}, \sqrt[3]{2}e^{\frac{-5\pi}{9}}$]

giving your answers in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [6]

(ii) Illustrate your values from b (i) on an Argand diagram. [3]

(c) The point *P* represents a complex number *z* on an Argand diagram, where

$$|z - 2| = \sqrt{2}|z - 4i|$$

Show that the locus of *P* is a circle, stating the coordinates of the centre and the radius of this circle. [7]
[Ans; $(x-2)^2 + (y-8)^2 = 40$]

Total: 21 marks

END OF EXAMINATION