PURE MATHEMATICS PREVIEW
UNIT 2 – TEST 1
1 hour 20 minutes

1. A botanist is studying the regeneration of an area of moorland following a fire. The total biomass in the area after t years is denoted by M tonnes and two models are proposed for the growth of M.

   Model A is given by

$$M = 900 - \frac{1500}{3t + 2}$$

   Model B is given by

$$M = 900 - \frac{1500}{2 + 5\ln(t + 1)}$$

   For each model, find

   (a) the value of M when $t = 4$ [2]

   (b) the rate at which the biomass is increasing when $t = 4$. [7]

   (c) Which model would regenerate the area of moorland faster? [1]

Total 10 marks

2. Given the experimental heat equation $u(x,t) = e^{-k^2 t}\sin x$

   (a) Find

   i. $\frac{\partial u}{\partial t}$ [1]

   ii. $\frac{\partial^2 u}{\partial x^2}$ [2]

   (b) Hence determine if the experimental equation satisfies the theoretical heat equation $\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2}$ [2]

Total 5 marks

3. Determine $\int_0^2 \tan^{-1} x$ [8]

4. (a) Copy and complete the table below for the equation $y = \frac{3\sin 2x}{2+\cos x}$. Give your answers to 5 decimal places. [2]

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.10819		0

(b) Use the trapezium rule, with all the values of y in the completed table, to solve

$$\int_0^{\frac{\pi}{2}} \frac{3\sin 2x}{2+\cos x}$$

giving your answer to 4 decimal places. [3]

(c) Using the substitution $u = 2 + \cos x$ show that

$$\int \frac{3\sin 2x}{2+\cos x} = 12\ln(2+\cos x) - 6\cos x - 12 + c$$ [5]

(d) Hence calculate the exact value of

$$\int_0^{\frac{\pi}{2}} \frac{3\sin 2x}{2+\cos x}$$ [2]

(e) State, to 2 significant figures, the difference between the exact value in (d) and the approximate value in (b). [1]

Total 13 marks

5. (a) Use DeMoivre's Theorem to prove that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

[6]

(b) Given the complex numbers $z_1 = 4 + 3i$, $z_2 = 3 + 4i$, $z_3 = a + bi$ where $a, b \in \mathbb{R}$

(i) Find the exact value of $|z_1 + z_2|$ in the form $x\sqrt{2}$. [4]

Given that $w = \frac{z_1 z_3}{z_2}$

(ii) find w in terms of a and b, giving your answer in the form $x + yi$, $x, y \in \mathbb{R}$ [6]

Given also that $w = \frac{2}{5} - \frac{-11}{5}i$

(iii) find the values of a and b. [6]

(iv) find $\arg w$, giving your answer in radians to 3 decimal places. [2]

Total 24 marks