1. (a) (i) Differentiate $f(x) = sin^{-1}3x$. [2]

(ii) Differentiate $y = \frac{\ln 2x}{tan^{-1}x}$ [3]

(b) The curve C is defined parametrically by

$$x = t(t-1), \quad y = \frac{4t}{1-t}\ ,\ t \neq 1$$

Find the equation of the tangent to the curve at $t = -1$ [6]

(c) Consider the curve defined by $4\cos x + \sin y = 3,$, show that $\frac{dy}{dx} = \frac{4\sin x}{cosy}$. [4]

(d) Let $(x, y) = 4x^2 + 3xy^2 + 7x + 3y$, find $\frac{\partial^2 f}{\partial x \partial y}$. [2]

Total: 17 marks

2. (a) Find $\int \frac{1}{\sqrt{5-4x-x^2}}\, dx$ [3]

(b) Let $f(x) = \frac{x^2+2x+3}{(x-1)(x^2+1)}$ $2 \leq x \leq 5$

i. Express $f(x)$ in partial fractions. [6]

ii. Hence find $\int_2^5 f(x)\ dx$. [4]

(c) It is given that for $n \geq 0$

$$I_n = \int_0^1 x^n e^{-x}\, dx$$

(i) Prove that for $n \geq 1, \quad I_n = nI_{n-1} - e^{-1}$. [3]

(ii) Find the exact value of I_3 . [4]

(d) Use the trapezium rule with 4 strips to find an approximation to

$$\int_3^7 2 - 3x^{\frac{1}{2}}\ dx$$

giving your answer to 2 decimal places. [4]

Total: 24 marks

3. (a) Showing all necessary working, express the complex number $\frac{1+3i}{4+2i}$ in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. [5]

(b) The complex number $2i$ is denoted by u.

It is given that u is root of the equation $x^3 + ax^2 + bx - 12 = 0$, where the constants a and b are real. Find the values of a and b. [5]

(c) The complex number u is given by $u = 8 + 6i$.

Find the two square roots of u.

Give your answers in the form $a + ib$, where a and b are exact. [5]

(d) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 - 5i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

Total: 19 marks