HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2018
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 – TEST 1 (PREVIEW)
1 hour 20 minutes

This examination paper consists of **2** printed pages.
This paper consists of **9** questions.
The maximum mark for this examination is **60**.

INSTRUCTIONS TO CANDIDATES
(i) Write your name clearly on each sheet of paper used
(ii) Answer **ALL** questions
(iii) Number your questions identically as they appear on the question paper and do **NOT**
   write your solutions to different questions beside each other
 (iv) Unless otherwise stated in the question, any numerical answer that is not <u>exact</u>, **MUST** be
   written correct to <u>three</u> (3) significant figures

EXAMINATION MATERIALS ALLOWED
(a) Mathematical formulae
(b) Scientific calculator (non-programmable, non-graphical)

1) Given that a and b are propositions, use the <u>algebra of propositions</u> to fully simplify
$(a \wedge b) \vee (\sim a \wedge b)$ [3]
Total: 3 marks

2) (a) (i) Express in terms of n, $\sum_{r=1}^{n+1}(2r)$. [3]
   (ii) Given that $\sum_{r=1}^{n+1}(2r) = 42$, find the possible value(s) of n. [4]

   (b) Determine the values of a and b such that $\dfrac{5\sqrt{3}-2}{3-\sqrt{3}} \equiv a + b\sqrt{3}$. [4]

Total: 11 marks

3) (i) Given $f(x) = x^3 + bx^2 + cx + 8$ is divisible by both $(x + 1)$ and $(x - 2)$, find the
   values of b and c. [4]

   (ii) Hence, with these values of b and of c, solve $f(x) = 0$. [4]

Total: 8 marks

4) Prove by mathematical induction that $\displaystyle\sum_{r=1}^{n} \frac{1}{(4r-3)(4r+1)} = \frac{n}{4n+1}$ $\forall\, n \in \textbf{\textit{N}}.$ [6]

Total: 6 marks

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5) (a) Solve for x, $2\log_2 x - \log_x 2 = 1$. [4]

(b) Solve for x the equation $e^x + 4e^{-x} = 4$, giving your answer in terms of *logs*. [4]

Total: 8 marks

6) The number of ants, N, on a tree initially was 30 000.

The number of ants after n days was found to be directly proportional to $\left(\dfrac{1}{2}\right)^n$.

Calculate estimates of

    (i) the number of ants after 5 days [2]
    (ii) the day on which the population is expected to reach 200. [4]

Total: 6 marks

7) (a) The function f is defined by $f: x \rightarrow ln\ (x - 4)$.

    (i)   Sketch the graph of f, showing clearly any intersection with the axes. [2]
    (ii) Determine an expression for the inverse function, $f^{-1}(x)$. [3]

(b) The function g is defined by $g: x \rightarrow e^x + 3$.
    Determine $gf(x)$, simplifying your answer. [2]

Total: 7 marks

8) Find the range of values of x for which $\dfrac{2x+3}{3x-2} + 2 \leq 0$, $x \neq \dfrac{2}{3}$. [5]

Total: 5 marks

9) Solve for $x \in R$, $2x^2 - |x| - 3 = 0$. [6]

Total: 6 marks

End of Test

SOLUTIONS – PREVIEW **CAPE 2018**: UNIT 1 TEST 1

1) b

2) (a) (i) $(n + 1)(n + 2)$ (ii) $n = 5$ (b) $a = \dfrac{3}{2}$, $b = \dfrac{13}{6}$

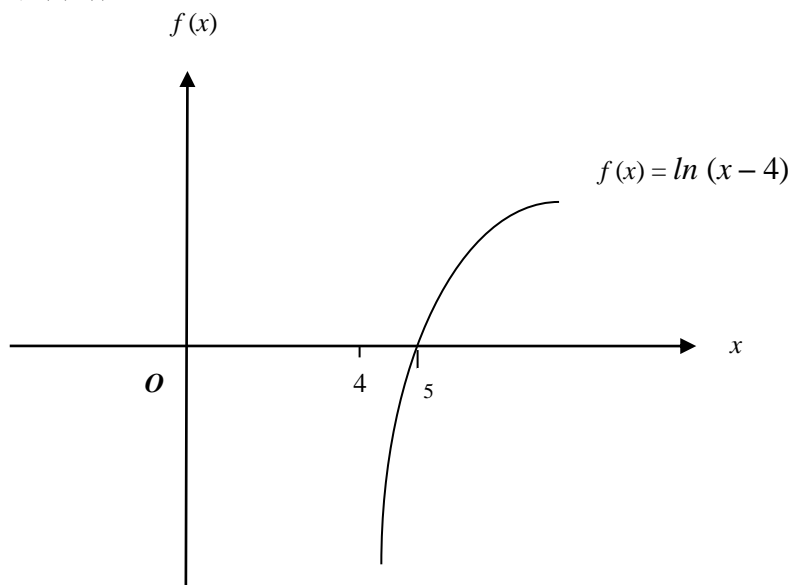**3)** (i) $b = -5$, $c = 2$ (ii) $x = -1$, $x = 2$, $x = 4$

5) (a) $p = \dfrac{3}{2}$, $p = -1$ (b) $x = \ln 2$

6) (i) 937.5 ants   (ii) 7.23 days $= n$

7) (a) (i)

(ii) $e^{x} + 4 = f^{-1}(x)$

   (b) $x - 1$

8) $\dfrac{1}{8} \le x < \dfrac{2}{3}$

9) $x = \pm \dfrac{3}{2}$