

HARRISON COLLEGE INTERNAL EXAMINATION MARCH 2016
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
SCHOOL BASED ASSESSMENT
PURE MATHEMATICS
UNIT 1 – TEST 1 (PREVIEW)
1 hour 20 minutes

This examination paper consists of 2 printed pages.
This paper consists of 9 questions.
The maximum mark for this examination is 60.

INSTRUCTIONS TO CANDIDATES

- (i) Write your name clearly on each sheet of paper used
- (ii) Answer **ALL** questions
- (iii) Number your questions identically as they appear on the question paper and do **NOT** write your solutions to different questions beside each other
- (iv) Unless otherwise stated in the question, any numerical answer that is not exact, **MUST** be written correct to three (3) significant figures

EXAMINATION MATERIALS ALLOWED

- (i) Mathematical formulae
- (ii) Scientific calculator (non-programmable, non-graphical)

1) Given that p and q are propositions, use the algebra of propositions to simplify fully
 $(p \wedge q) \vee (p \wedge \sim q)$ [3]

2) (i) Express in terms of n , $\sum_{r=1}^{n+1}(r + 2)$. [4]
(ii) Given that $\sum_{r=1}^{n+1}(r + 2) = 7n$, find the possible value(s) of n . [5]

Ans. $n = 1$ or $n = 6$

3) Given that -1 and 2 are two roots of the equation $x^3 + px^2 + x + q = 0$.
(i) Find the values of p and q [6]
(ii) Hence or otherwise, find the other root of the equation. [3]
Ans. $p = -4$, $q = 6$; $x = 3$

4) Prove by mathematical induction that $\sum_{r=1}^n \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1} \forall n \in \mathbb{Z}^+$. [7]

5) Solve for $y \in \mathbb{R}$, the equation $6e^{-2y} = 7e^y + 3$, giving your answer in terms of logs. [5]
Ans. $\ln\left(\frac{3}{2}\right)$

6) The population, $P(n)$, of a town at the beginning of the year 2000 was 2400. The population increased so that, after a period of n years, the new population was found to be directly proportional to $(1.06)^n$. Calculate estimates of

(i) the population at the beginning of 2010. [3]

(ii) the year in which the population is expected to first reached 7000. [4]

Ans. 4298; 2018

7) The function f is given by $f: x \rightarrow e^{2x}$, $x \in \mathbf{R}$ and the function g is given by $g: x \rightarrow \ln 2x$, $x \in \mathbf{R}$, $x > 0$.

(i) Sketch the graph of f , showing clearly any intersection with the axes. [2]

(ii) Find an expression in terms of x for $f^{-1}(x)$. [4]

(iii) State for $f^{-1}(x)$

(a) the domain [1]

(b) the range. [1]

(iii) Determine $fg(x)$, simplifying your answer. [3]

Ans. $\frac{1}{2} \ln x = f^{-1}(x)$; $x \in \mathbf{R}$, $x > 0$; $f(x) \in \mathbf{R}$; $4x^2$

8) Find the range of values of $x \in \mathbf{R}$ for which $\frac{x-2}{x-3} \leq 0$, $x \neq 3$. [4]

Ans. $2 \leq x < 3$

9) Find the range of values of x such that $|4 - 3x| \leq x$. [5]

Ans. $1 \leq x \leq 2$

End of Examination