

Preview test

1. A sequence is given by

$$x_1 = 1$$

$$x_{n+1} = x_n(p + x_n)$$

where p is a constant ($p \neq 0$).

- (i) Find x_2 in terms of p .
(ii) Show that $x_3 = 1 + 3p + 2p^2$

Given that $x_3 = 1$,

- (iii) find the value of p .

2. A boy saves some money over a period of 60 weeks. He saves \$10 in week 1, \$15 in week 2, \$20 in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

- a) Find how much he saves in week 15
b) Calculate the total amount he saves over the 60 week period.

3. A series of positive integers u_1, u_2, u_3, \dots is defined by

$$u_1 = 2 \text{ and } u_{n+1} = 5u_n - 4, \text{ for } n \geq 1.$$

Prove by induction that $u_n = 5^{n-1} + 1$, for $n \geq 1$.

4. a) Express $\frac{1}{r(r+2)}$ in partial fractions.

- b) Hence prove, by the method of differences, that

$$\sum_{r=1}^n \frac{4}{r(r+2)} = \frac{n(an+b)}{(n+1)(n+2)}$$

- c) Find, to 4 decimal places, the value of

$$\sum_{r=51}^{100} \frac{4}{r(r+2)}$$

5. a) Write down and simplify the first three non-zero terms of the Maclaurin series for $\ln(1 + 2x)$.

- b) Hence find the first three non-zero terms of the Maclaurin series for $e^{2x}\ln(1 + 2x)$, simplifying the coefficients.

6. a) Use the binomial series to expand $\frac{1}{\sqrt[3]{8-5x}}$, where $|x| < \frac{8}{5}$, in ascending powers of x up to and including the term in x^2 . Simplify each term.

b) Hence, or otherwise, find the first 3 terms in the expansion of $\frac{x+2}{\sqrt[3]{8-5x}}$ as a series in ascending powers of x .

7.

$$f(x) = 3x^2 + x - \tan\left(\frac{x}{2}\right) - 2$$

a) Show that the equation $f(x) = 0$ has a root α between 0.7 and 0.8

b) Use linear interpolation once, to obtain an approximation to α . Give your answer to 3 decimal places.

c) Taking 0.75 as your first approximation to α , apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α . Give your answer to 3 decimal places.

Answers

Question 5

Question 1

a) $2x - 2x^2 + \frac{8}{3}x^3$

b) $2x + 2x^2 + \frac{8}{3}x^3$

(i) $x_2 = p + 1$

(iii) $p = -\frac{3}{2}$

Question 6

Question 2

a) \$80

b) \$9450

a) $\frac{1}{2} + \frac{5}{48}x + \frac{25}{576}x^2$

b) $1 + \frac{17}{24}x + \frac{55}{288}x^2$

Question 4

a) $\frac{1}{2r} - \frac{1}{2(r+2)}$

b) $\frac{n(3n+5)}{(n+1)(n+2)}$

c) 0.0383

Question 7

b) 0.740

c) 0.741